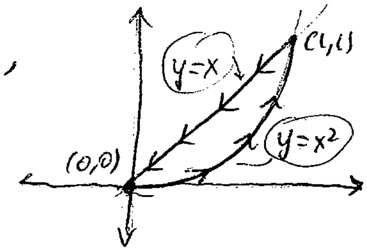
Quiz 6

Name: key

[7] **1.** Evaluate $\int_C (3x^2+y)\,dx + 2xy^2\,dy$, where C is the curve which starts at $(0,0)$, proceeds to $(1,1)$ along the graph of $y=x^2$, and then returns to $(0,0)$ along a straight line (see diagram).



→ For bottom portion of curve: $\begin{cases} x=t \\ y=t^2 \end{cases}$ (1), $\begin{cases} dx=dt \\ dy=2t\,dt \end{cases}$ (1), $0\le t\le 1$,

get $\int_0^1 (3t^2+t^2)\,dt + 2t(t^2)^2(2t\,dt) = \int_0^1 (4t^2+4t^6)\,dt$

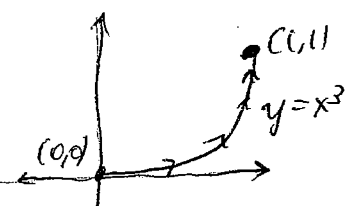
$= \left[\frac{4t^3}{3}+\frac{4t^7}{7}\right]_0^1 = \frac{4}{3}+\frac{4}{7} = \frac{40}{21}.$ (1)

→ For top portion of curve: $\begin{cases} x=t \\ y=t \end{cases}$. $\begin{matrix} dx=dt \\ dy=dt \end{matrix}$ (1), $0\le t\le 1$, we get (1)

$\int_0^1 (3t^2+t)\,dt + 2t\,t^2\,dt = \int_0^1 (3t^2+t+2t^3)\,dt = \left[t^3+\frac{t^2}{2}+\frac{t^4}{2}\right]_0^1 = 1+\frac{1}{2}+\frac{1}{2} = 2.$

– So the integral over the whole curve C is (2) $\frac{40}{21}-2 = \boxed{\frac{-2}{21}}$.

[6] **2.** Evaluate $\int_C y\,ds$, where C is the curve which starts at $(0,0)$ and proceeds to $(1,1)$ along the graph of $y=x^3$.

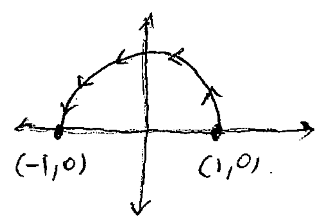


Here $\begin{cases} x=t \\ y=t^3 \end{cases}$ (1), $\begin{cases} dx=dt \\ dy=3t^2\,dt \end{cases}$, $ds=\sqrt{dx^2+dy^2}$ (1) $=\sqrt{1+9t^4}\,dt$ (1),

so $\int_C y\,ds = \int_{t=0}^{t=1} t^3\sqrt{1+9t^4}\,dt$. Take $\begin{matrix} u=1+9t^4 \\ du=36t^3\,dt \end{matrix}$ (1),

to get $\int_{u=1}^{u=10} \frac{1}{36}\sqrt{u}\,du$ (1) $= \frac{1}{36}\left[\frac{u^{3/2}}{3/2}\right]_1^{10}$ (1) $= \boxed{\frac{1}{54}\left(10^{3/2}-1\right)}$

[7] **3.** Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = (x^2+y^2)\mathbf{i} + 2xy\mathbf{j}$ and C is the the upper half of the unit circle, traversed in a counterclockwise direction from $(1,0)$ to $(-1,0)$ (see diagram).



Here $\begin{cases} x=\cos t \\ y=\sin t \end{cases}$ (1), $\begin{cases} dx=-\sin t\,dt \\ dy=\cos t\,dt \end{cases}$ (1), $0\le t\le \pi$,

so $\int_C \vec{F}\cdot d\vec{r} = \int_C (x^2+y^2)\,dx + 2xy\,dy$ (1) $=$

$= \int_0^\pi (\cos^2 t+\sin^2 t)(-\sin t\,dt) + 2\cos t\sin t\cos t\,dt$ (1) $= \int_0^\pi \left[-\sin t + 2\cos^2 t\sin t\right]dt$

$= \left[\cos t - \frac{2\cos^3 t}{3}\right]_0^\pi$ (1) (1) $= \left[(\cos\pi-\cos 0) - \frac{2}{3}\left(\cos^3\pi - \cos^3 0\right)\right] = (-1-1)-\frac{2}{3}(-1-1)$ (1)

$= \boxed{-2/3}$.