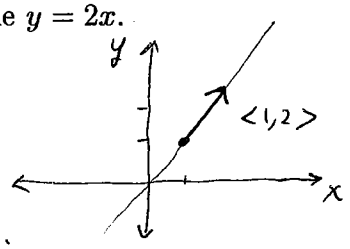
Quiz 4

Name: key

1. A curve C is the intersection of the surface $z = x^2 + y^2$ with the plane $y = 2x$. Find the slope of C at the point $P(1, 2, 5)$. [6]



The vector $\langle 1,2\rangle$ is parallel to the line $y = 2x$ in the xy-plane. A unit vector parallel to $\langle 1,2\rangle$ is $\vec{u} = \frac{\langle 1,2\rangle}{|\langle 1,2\rangle|} = \frac{\langle 1,2\rangle}{\sqrt{1^2+2^2}} = \frac{1}{\sqrt{5}}\langle 1,2\rangle = \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$. (2)

The slope of C at P is $D_{\vec{u}} z|_P = \vec{\nabla} z|_P \cdot \vec{u} = \langle 2x, 2y\rangle|_P \cdot \vec{u} =$ (2)

$= \langle 2\cdot 1, 2\cdot 2\rangle \cdot \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle = \frac{2}{\sqrt{5}} + \frac{8}{\sqrt{5}} = \frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}.$ (2)

2. Let $f(x, y, z) = x^2 + 2y^2 + 7z^2$.

a) Find ∇f at the point $P(5, 2, 1)$. [7]

$\langle 2x, 4y, 14z\rangle|_P = \langle 10, 8, 14\rangle.$ (2) (1)

b) Find an equation of the tangent plane to the level surface of f through P.

$10(x-5) + 8(y-2) + 14(z-1) = 0.$ (4)

3. Suppose R is the rectangle in the xy-plane given by $0 \le x \le 1$, $2 \le y \le 5$. Find the value of $\int\int_R (e^x + y^2)\, dA$, showing all work. [7]

$$\int_2^5 \int_0^1 (e^x + y^2)\, dx\, dy = \int_2^5 \left[e^x + xy^2 \right]_{x=0}^{x=1} dy =$$ (1) (1) (1)

$$= \int_2^5 \left[(e^1 + y^2) - (e^0 + 0) \right] dy = \int_2^5 (e^1 - 1 + y^2)\, dy =$$ (1)

$$= \left[(e-1)y + \frac{y^3}{3} \right]_{y=2}^{y=5} = \left((e-1)\cdot 5 + \frac{125}{3} \right) - \left((e-1)\cdot 2 + \frac{8}{3} \right)$$ (1)

$$= 3(e-1) + \frac{117}{3} = 3e + 36.$$