Quiz 3

Name: Key

1. A particle moves through space in such a way that its position vector at time t is given by $\langle \ln t, 2t, t^2 \rangle$.

a) Find the velocity vector at time $t = 1$.

[2] $\vec{r}'(t) = \langle \frac{1}{t}, 2, 2t \rangle$; so $\vec{r}'(1) = \langle 1, 2, 2 \rangle$

b) Find the speed at time $t = 1$.

[2] $|\vec{r}'(1)| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1+4+4} = 3$

c) Find the acceleration vector at time $t = 1$.

[2] $\vec{r}''(t) = \langle -\frac{1}{t^2}, 0, 2 \rangle \Rightarrow \vec{r}''(1) = \langle -1, 0, 2 \rangle$

d) Is the acceleration vector perpendicular to the velocity vector? Explain how you know.

[2] No, because $\vec{r}' \cdot \vec{r}'' \neq 0$. ($\vec{r}' \cdot \vec{r}'' = \langle 1,2,2 \rangle \cdot \langle -1,0,2 \rangle = 1\cdot(-1) + 2\cdot 0 + 2\cdot 2 = 3$.)

2. Suppose a particle moves in such a way that its velocity vector is always parallel to its velocity vector. Explain why it follows that the curvature of the path followed by the particle must be zero. (Hint: $\kappa = \frac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3}$.)

[4] Since $\vec{r}'$ is parallel to $\vec{r}''$, then the angle θ between them is zero, (2) so $|\vec{r}' \times \vec{r}''| = |\vec{r}'||\vec{r}''| \sin\theta = 0$, so $\kappa = 0$. (2)

3. For the function $f(x, y) = \dfrac{x}{x^2 + y^2}$, compute

a) $\frac{\partial f}{\partial x}$

[4] $$= \frac{(x^2+y^2)\frac{\partial}{\partial x}(x) - x\frac{\partial}{\partial x}(x^2+y^2)}{(x^2+y^2)^2} \quad (2) \quad = \frac{(x^2+y^2)\cdot 1 - x \cdot 2x}{(x^2+y^2)^2} \quad (2)$$

$$= \boxed{\frac{y^2 - x^2}{(x^2+y^2)^2}}$$

b) $\frac{\partial f}{\partial y}$

[4] $$= \frac{(x^2+y^2)\frac{\partial}{\partial y}(x) - x\frac{\partial}{\partial y}(x^2+y^2)}{(x^2+y^2)^2} \quad (2) \quad = \frac{(x^2+y^2)\cdot 0 - x \cdot 2y}{(x^2+y^2)^2} \quad (2)$$

$$= \boxed{\frac{-2xy}{(x^2+y^2)^2}}$$