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Capstone Project

Objective: To show that, given a circle with a certain area, it is impossible to construct a a square using only a straight edge and compass with the same area as the given circle.

Definitions:

Algebraic Number: A number, α, is algebraic if some combination of powers of α with integer coefficients is rational.

Constructible: Any number that can be found by a whole number and using addition, subtraction, division, multiplication, and root.

Transcendental: Any number that is not algebraic

Proofs: To prove the objective, it must be shown that Π is transcendental. The following proofs will lead to the proof that Π is transcendental.

Lindemann–Weierstrass theorem consequence: If α is algebraic, then e^α is transcendental. The Lindemann-Weierstrass theorem is very complex and time consuming and rather than proving the theorem, we will simply take it to be true.

Proof #1: If x is algebraic, then (1/x) is algebraic. (Proof attached)

Proof #2: If P1, P2 are algebraic, then (P1*P2) is also algebraic. (Proof attached)

Proof #3: That Π is transcendental using Euler's identity. (Proof attached)

Conclusion: We will show how, if Π is transcendental, it follows that it is impossible to square a circle.

Bibliography

Hardy, G.H. *Pure Mathematics*. New York: Cambridge Press, 1960.

"Euler's Identity." *Wikipedia.com* <http://en.wikipedia.org/wiki/Euler_identity.>

"Lindemann-Weierstrass Theorem." *Wikipedia.com* *<http://en.wikipedia.org/wiki/Lindemann-Weierstrass_theorem>*

Proof #1

Claim. If x is algebraic, then $\frac{1}{x}$ is algebraic.

Proof: If x is algebraic, then by definition:

$$a_0 + a_1x + a_2x^2 + \cdots + a_nx^n = 0.$$

Dividing by x^n: $\frac{a_0}{x^n} + \frac{a_1}{x^{n-1}} + \frac{a_2}{x^{n-2}} + \cdots + a_n = 0$

$$= a_0\left(\frac{1}{x}\right)^n + a_1\left(\frac{1}{x}\right)^{n-1} + a_2\left(\frac{1}{x}\right)^{n-2} + \cdots + a_n = 0$$

let $W = \frac{1}{x}$: $\underline{a_0W^n + a_1W^{n-1} + a_2W^{n-2} + \cdots + a_n = 0}$

This takes the form given in the definition of algebraic.
so, W, or $\frac{1}{x}$, is algebraic. $\boxed{\text{QED}}$

Proof #2

Claim: The product of two algebraic numbers is itself algebraic.

Proof: Let P_1, P_2 be algebraic numbers s.t. $P_1 \cdot P_2 = \beta$

so, $\frac{1}{P_1}\beta = P_2 \Rightarrow \frac{1}{P_1}\beta - P_2 = 0$.

Let $\alpha_0 = -P_2$ and $\alpha_1 = \frac{1}{P_1}$.

we then get the equation: $\alpha_0 + \alpha_1\beta = 0$.

So, β must be algebraic because it is a solution to this equation. The equation is also in the form of $a_0 + a_1x = 0$, where x is algebraic by definition.

So, β, the product of two algebraic numbers is algebraic as well. $\boxed{QED}$

Proof #3

Claim: π is transcendental.

Proof (by contradiction): Suppose π is not transcendental, then π is algebraic. We know i is algebraic because i satisfies the equation $x^2 = -1$. Using proof #2, we know that if i is algebraic and π is algebraic, then πi must also be algebraic. Also, e^{a} is transcendental if a is algebraic. But, by euler's identity, $e^{\pi i} = -1$. But -1 is not transcendental because it is obviously algebraic, so this is a contradiction. QED