Wasan and Sangaku
Japanese Temple Geometry

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Throughout the Japanese Tokugawa period from 1600-1868, the country of Japan practiced a strict habit of self isolation. During this era all contact with foreign influences was rigorously prohibited, except for a few contacts with Dutch traders in limited regions. The lack of relations with others countries allowed Japanese culture to flourish. One result of this isolation is Wasan, or "Japanese mathematics." Created separately from western European mathematics, there are still similarities between the different forms: Japanese wasan still contemplates a great deal of Euclidean geometry. However, there are some differences in Japanese geometry, mainly that great emphasis is placed on the study of close packing of circles into larger figures. Even still, some similarities can be seen between wasan schools and the Pythagoreans: both developments discuss the profound relationship between math and art. Wasan is an integral part of Japanese culture and offers some insight into Japanese mathematics, religion, and art.

A particular feature of wasan geometry that highlights the cultural association between numbers and art is sangaku, which literally means "mathematical tablet". A common practice during the Tokugawa administration was to offer geometric problems and solutions inscribed on wood tablets to the gods. The custom of hanging tablets at shrines was established in Japan centuries before sangaku was developed; however some of the oldest surviving tablets of sangaku date from 1683. Devotees of math, most likely samurai, farmers, and merchants would solve an assortment of geometry problems and present them on delicately colored wooden tablets
which would then be hung under the roofs of religious buildings. Usually only the result of the theorem was given, not the proof. Admirers could enjoy the beauty of these tablets or could attempt to solve a problem themselves.

In 1868 the Tokugawa regime ended and was replaced by the Meiji period during which the country of Japan opened up to influences from other countries. Since wasan had never been used to describe any phenomena of nature, it seemed unpractical to continue the tradition.\[1872 - Ministry of Education orders that state schools stop teaching wasan a decade later, it had slipped into obscurity\]

Japanese math only described static systems, not dynamic ones, so the first translations of western works were done using Japanese philosophical terms instead of wasan terms. Thus western math practices, which were being applied to things like physics, mechanics and other scientific realms, were adopted in Japan.

Sangaku Presentation Bibliography
MATH 4513
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___: “Incenter”
___: “Incircle.”
OUTLINE

1. Tokugawa Regime in Japan
   a) self-isolation 1600-1868
   b) culture flourishes
   c) wasan created

2. Wasan
   a) different than Western math
   b) Euclidean geometry
   c) Emphasis on spheres and circles
   d) Wasan schools similar to Pythagorean schools

3. Sangaku “mathematical tablet”
   a) division of wasan
   b) reflective of Japanese culture
   c) art, religion, and mathematics
   d) merchants, farmers, samurai class
   e) geometric theorems inscribed on wooden tablets
   f) colorful and detailed
   g) show result, not proof
   h) hang from the roofs of religious buildings
   i) admirers

4. Proof of triangle problem

5. Proof of Casey’s theorem
What is the relationship of the radii of the circles?

First observe that all of the minor triangles are similar to each other.

If we label one triangle with angles $x, y$, and $L$ (right $L$) then by complementary angles it is easy to label all angles of every triangle, including the biggest one.

Second we observe that each triangle area can be expressed in terms of the radius of the incircle, and from this we find that if the triangles are similar by some ratio $k$, then the radii are also similar by the ratio $k$.

Each triangle can be expressed as the sum of 3 smaller triangles (the area can be expressed) that are made of one side of the large $\Delta$ and each have a height $r$. Also, $r$ is then the radius of the "incircle" of the large triangle.

So $\Delta_{ABC} = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr = \frac{1}{2}r(a + b + c)$, but $S = \frac{a+b+c}{2}$ so Area $\Delta_{ABC} = r \sqrt{S}$ where $r_i$ is the radius of the incircle of $\Delta_i$.

Take two right triangles (since we are only dealing with right triangles in this problem) that are similar,

\[
\frac{a_1}{a_2} = \frac{b_1}{b_2} = k \quad \text{so} \quad a_2 = ka_1 \text{ and } b_2 = kb_2 \text{ so Area } \Delta_2 = \frac{1}{2}ka_1kb_1 = \frac{1}{2}k^2a_1b_1
\]

Now Area ($a_i$) of triangle 2: $A_2 = k^2A_1 = r_2S_2$

$\frac{k^2r_1s_1}{r_2s_1} = k$ so the radii are comparable by some ratio $k$ that is the same ratio for comparing the triangles.

$k = \frac{r_2}{r_1}$
Casey's Theorem

What is the relationship between \( r_1, r_2, r_3, r_4 \) and \( \alpha \)?

Casey's Thm: \( T_{12} T_{34} - T_{13} T_{42} + T_{14} T_{23} = 0 \)

Obviously, 4 of the 6 T's are:

\[ T_{23} = \alpha - r_2 - r_3 \]
\[ T_{14} = \alpha - r_1 - r_4 \]
\[ T_{12} = \alpha - r_1 - r_2 \]
\[ T_{34} = \alpha - r_3 - r_4 \]

To find the other two, we use the Pythagorean theorem:

\[ ALO6 \quad r_3 > r_1. \]

\[ T_{13}^2 + (r_3 - r_1)^2 = (\alpha - r_1 - r_3)^2 + (\alpha - r_1 - r_3)^2 \]

\[ T_{13}^0 = \sqrt{2(\alpha - r_1 - r_3)^2 - (r_3 - r_1)^2} \]

Similarly, \( T_{24}^0 = \sqrt{2(\alpha - r_2 - r_4)^2 + (r_4 - r_2)^2} \); if we have \( ALO6 \), \( r_4 > r_2 \).