How to set up a double integral over a region R in the xy-plane

1. Find equations for the curves which bound R.
2. For each of these equations there is an inequality which determines the side of the corresponding curve that R lies on.
3. Choose the order you want to do the iterated integral in: either integrating first with respect to x and then with respect to y, or first with respect to y and then with respect to x. Let's suppose you choose to integrate with respect to x first. (If you choose to integrate with respect to y first, switch the roles of x and y in what follows).
4. Find the largest y-coordinate of any point in R and the smallest y-coordinate of any point in R. Call the largest y-coordinate d and the smallest y-coordinate d.
5. Write down the outer integral, with respect to y. It will be from $y = c$ to $y = d$.
6. Each value of y between c and d corresponds to a horizontal line in the xy-plane, whose points have that value of y as their y-coordinate. On each such line, the points which lie in R are those whose x-coordinates satisfy the inequalities found in step 2 above. Use these inequalities to write down the inner integral, with respect to x. The endpoints of the inner integral will be functions of y.

How to set up a triple integral over a region E in xyz-space.

(Note: you will probably never use the procedure actually given below to set up a triple integral. In most cases, setting up the integral will be somewhat simpler. But it's still useful to understand the general procedure.)

1. Find equations for the surfaces which bound E.
2. For each of these equations there is an inequality which determines the side of the corresponding curve that E lies on.
3. Choose the order you want to do the iterated integral in: either integrating first with respect to x, then with respect to y, and then with respect to z; or first with respect to x, then with respect to z, and then with respect to y; or first with respect to y, then with respect to x, and then with respect to z; ... (there are six possible orders to choose from). Let's suppose you choose to integrate first with respect to x, then y, then z. (If you choose another order, the variables below have to be switched accordingly.)
4. Find the largest z-coordinate of any point in E and the smallest z-coordinate of any point in E. Call the largest z-coordinate d and the smallest z-coordinate c.
5. Write down the outer integral, with respect to z. It will be from $z = c$ to $z = d$.
6. Each value of z between c and d corresponds to a horizontal plane in xyz-space, whose points have that value of z as their z-coordinate. On each plane, the points which lie in E form a two-dimensional region R_z in the xy-plane. (Actually, R_z lies not in the xy-plane but in a horizontal plane a distance z above the xy-plane, but we can still think of R_z as being located in the xy-plane because the locations of points in R_z are determined by their x and y coordinates. We are calling the region R_z instead of just R to indicate that the region varies with z.) The boundaries of R_z are curves in the xy-plane which are determined by the same equations as in step 1, except now z is fixed and only x and y can vary.
7. Find the largest y-coordinate of any point in R_z, and call it f. Find the smallest y-coordinate of any point in R_z, and call it e. Since the values of e and f will change when z changes, e and f are really functions of z, so we write them as $e(z)$ and $f(z)$.
8. Write down the middle integral, with respect to y. It will be from $y = e(z)$ to $y = f(z)$.
9. Each value of y between $e(z)$ and $f(z)$ corresponds to a line in xyz-space whose points have that value of y as their y-coordinate, and which lies in a horizontal plane z units above the xy-plane. (The line is therefore parallel to the x-axis, because all points on the line have the same y and z coordinates.) On each such line, the points which lie in E are those whose x-coordinates satisfy the inequalities found in step 2 above. Use these inequalities to write down the inner integral, with respect to x. The endpoints of the inner integral will be functions of y and z.