Calculus of Variations: Review for Midterm

Definition of a normed space, strong and weak norms on $C^1[a,b]$.
Definition of strong and weak relative minima.

Definition of first variation and second variation of a functional on a normed space.

A necessary condition for a functional to have a weak relative minimum is that $\delta J[h] = 0$ and $\delta^2 J[h] \geq 0$ for all admissible variations. (Why is this so?)

Length functional on surfaces in R^3. (Minimizers of length functional are called "geodesics".)

Fundamental lemma of calculus of variations.
Du bois-Raymond's lemma.

A necessary condition for $J[y] = \int_a^b F(x,y,y')\,dx$ to have a weak relative minimum at $\hat{y}$ is that $\hat{y}$ be a solution of the Euler-Lagrange equation $F_y - \frac{d}{dx}F_{y'} = 0$ on $[a,b]$.

Examples of solving the Euler-Lagrange equation and using the given boundary conditions to determine possible relative minimizers.
How to integrate the Euler-Lagrange equation in the case when F is independent of y, or when F is independent of x.

Form of the Euler-Lagrange equation when $J[y] = \int_a^b F(x,y,y',y'')\,dx$.
Form of the Euler-Lagrange equation when J depends on two functions $y(x)$ and $z(x)$.
Examples of solving the Euler-Lagrange equation in these cases.

Form of the Euler-Lagrange equation when J depends on a function $z(x,y)$ of two variables x and y. (We haven't solved any Euler-Lagrange equations in this case yet; because they are partial differential equations, and significantly more complicated to solve than the ordinary differential equations we've been solving.)

Form of the Euler-Lagrange equation for the problem of minimizing $J[y] = \int_a^b F(x,y,y')\,dx$ subject to the condition that $K[y] = \int_a^b G(x,y,y')\,dx$ be held constant. This is called an isoperimetric constraint.
Form of the Euler-Lagrange equation for the problem of minimizing $J[y,z] = \int_a^b F(x,y,z,y',z')\,dx$ subject to the condition that $g(x,y(x),z(x)) = 0$ for all $x \in [a,b]$. This is called a pointwise holonomic constraint.
We solved the Euler-Lagrange equations for a couple of constrained variational problems in class.

If we are minimizing $J[y] = \int_a^b F(x,y,y')\,dx$ and we do not require y to have a fixed value at a, then we are said to have a "free boundary condition" at a. In that case, a necessary requirement for a minimizer is that it satisfy the "natural boundary condition" $F_{y'} = 0$ at $x = a$. Similarly, we can have a free boundary condition at b, and a natural boundary condition for minimizers at b. The variational problem could have a fixed boundary condition at both endpoints, or a fixed boundary condition at one endpoint and a free condition at the other, or free conditions at both endpoints. In any case there will be two corresponding boundary conditions that minimizers must satisfy. (This material is covered in section 9.1 of Kot's book.)

The material we have discussed in the past couple of weeks (Legendre's and Jacobi's theory of the second variation, and sufficient conditions for a weak extremum) will not be covered on the midterm.