

$$\left[\begin{array}{ccc|ccc} \underbrace{3 & 0 & 4}_A & & \underbrace{1 & 0 & 0}_{I_3} \\ 4 & 1 & 3 & & 0 & 1 & 0 \\ 3 & 2 & 0 & & 0 & 0 & 1 \end{array} \right]$$

$$\downarrow \text{div } R_1 \text{ by } 3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{4}{3} & \frac{1}{3} & 0 & 0 \\ 4 & 1 & 3 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$\downarrow \begin{array}{l} R_2 - 4 \cdot R_1 \rightarrow R_2 \\ R_3 - 3 \cdot R_1 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{4}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{7}{3} & -\frac{4}{3} & 1 & 0 \\ 0 & 2 & -4 & -1 & 0 & 1 \end{array} \right]$$

$$\downarrow R_3 - 2 \cdot R_2 \rightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{4}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{7}{3} & -\frac{4}{3} & 1 & 0 \\ 0 & 0 & +\frac{2}{3} & \frac{5}{3} & -2 & 1 \end{array} \right]$$

$$\downarrow R_3 \text{ div by } \frac{2}{3} \rightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{4}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{7}{3} & -\frac{4}{3} & 1 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & -3 & \frac{3}{2} \end{array} \right]$$

$$\downarrow \begin{array}{l} R_2 - (-\frac{7}{3})R_3 \rightarrow R_2 \\ R_1 - (\frac{4}{3})R_3 \rightarrow R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & \frac{9}{2} & -6 & \frac{7}{2} \\ 0 & 0 & 1 & \frac{5}{2} & -3 & \frac{3}{2} \end{array} \right]$$

$$\underbrace{\quad \quad \quad}_{I_3} \quad \quad \quad \underbrace{\quad \quad \quad}_{A^{-1}}$$

Procedure for finding the inverse of $A = \begin{bmatrix} 3 & 0 & 4 \\ 4 & 1 & 3 \\ 3 & 2 & 0 \end{bmatrix}$.

The result is: $A^{-1} = \begin{bmatrix} -3 & 4 & -2 \\ \frac{9}{2} & -6 & \frac{7}{2} \\ \frac{5}{2} & -3 & \frac{3}{2} \end{bmatrix}$

$$E_1 = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 3 & 0 & 4 \\ 4 & 1 & 3 \\ 3 & 2 & 0 \end{bmatrix}$$

$$E_1 A = \begin{bmatrix} 1 & 0 & \frac{4}{3} \\ 4 & 1 & 3 \\ 3 & 2 & 0 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_2 E_1 A = \begin{bmatrix} 1 & 0 & \frac{4}{3} \\ 0 & 1 & -\frac{7}{3} \\ 3 & 2 & 0 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$E_3 E_2 E_1 A = \begin{bmatrix} 1 & 0 & \frac{4}{3} \\ 0 & 1 & -\frac{7}{3} \\ 0 & 2 & -4 \end{bmatrix}$$

etc.

$$E_7 E_6 E_5 E_4 E_3 E_2 E_1 A = I_3$$

$$A = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} E_5^{-1} E_6^{-1} E_7^{-1}$$