Math 4433
Exam 3

1. *(20 points)*
a. Define "f is continuous at c".
b. Prove that the composition of continuous functions is continuous.

2. *(20 points)*
a. Define "the derivative of f at c is L".
b. Prove that if a function is differentiable at a point, then it is continuous at that point.

3. *(20 points)* Suppose $f : \mathbf{R} \to \mathbf{R}$ is continuous on $\mathbf{R}$, and suppose that for all $x \in \mathbf{Q}$, $f(x) \geq 0$. Show that for all $x \in \mathbf{R}$, $f(x) \geq 0$.

4. *(20 points)*
a. Suppose $f : [a, b] \to \mathbf{R}$ is continuous on $[a, b]$. Suppose that for all $x \in [a, b]$, $f(x) \neq 0$. Show that $1/f$ is bounded on $[a, b]$.
b. Give an example of a function $f : [0, 1] \to \mathbf{R}$ which is continuous on $[0, 1]$, but $1/f$ is not bounded on $[0, 1]$.

5. *(20 points)* Define $f(x)$ for all $x \neq 0$ by $f(x) = 1/x^2$. Use the definition of derivative to show that for all $c \neq 0$, $f'(c) = -2/c^3$.

6. *(20 points)* Define $f : \mathbf{R} \to \mathbf{R}$ by

$$f(x) = \begin{cases} x \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Show that f is not differentiable at 0.