Math 4433
Test 2

1. *(15 points)* Define the sequence (s_n) by $(s_n) = (0, 1, 0, 2, 0, 3, 0, 4, 0, 5, \dots)$. Is it true that $\lim s_n = \infty$? Prove your answer.

2. *(20 points)* Prove that $\lim(3^n + 5^n)^{1/n} = 5$.
   (Hint: use the fact that $5^n \leq 3^n + 5^n \leq 2 \cdot 5^n$.)

3. *(30 points)* Suppose $s_1 = 1$ and $s_{n+1} = \sqrt{2s_n}$ for all n. Prove that s_n converges, and find the limit.

4. *(15 points)* Suppose (s_n) and (t_n) are sequences such that for all m and n, $|s_m - s_n| \leq |t_m - t_n|$. Prove that if (t_n) is convergent then (s_n) is convergent.

5a. *(10 points)* Show that if S_1 and S_2 are bounded then $S_1 \cup S_2$ is bounded.
5b. *(10 points)* Show that if S_1 and S_2 are compact then $S_1 \cup S_2$ is compact.