Calculations:

An example of Random-Matrix multiplications is: what is $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n$?

1st we choose different values for n and see if there is a pattern:

For $n=1$, we get: $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$n=2$ $\quad \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$n=3$ $\quad \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$

$n=4$ $\quad \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

$n=5$ $\quad \begin{bmatrix} 8 & 5 \\ 5 & 3 \end{bmatrix}$

These multiplications lead to the following order:

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$$

where F_n is a Fibonacci number.

And then we prove by Induction:

Let's Assume that $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$ is true

And prove that $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n+1} = \begin{bmatrix} F_{n+2} & F_{n+1} \\ F_{n+1} & F_n \end{bmatrix}$

(we have to compute:) $\begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} F_{n+1} + F_n & F_{n+} \\ F_n + F_{n-1} & F_{n} \end{bmatrix}$

Wich gives us: $\begin{bmatrix} F_{n+2} & F_{n+1} \\ F_{n+1} & F_n \end{bmatrix}$ Thus, the Assumption is tru

The Identity $\begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$ is defined as the Q-Matrix

$Q^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$ this matrix immediatly gives some very important identities, for instance:

$(Q^{n+1}) * (Q^n) = Q^{2n+1}$

$(Q^m) \times (Q^{n-1}) = Q^{n+m-1}$ etc.

A number of Mathematicians wondered what could be the importance of Random-Matrix multiplications. According to Anderson these random multiplications lead to order somehow. For example if we take a semiconductor with some impurities and run a current through it, we do not expect the current to pass through because of these impurities but somehow it does go through.

In the case of seeing through glass we explained it as follows: Glass has an irregular molecular structure so we do not expect light to be seen through. If we look at every molecule individually and give it a matrix to represent it, every time the light ray hits a new molecule can be considered a random multiplication. At the end we see light through glass, as we all know, instead of a blurry image which leads us to believe that Random-Matrix multiplications lead to order.

References:

http://en.wikipedia.org/wiki/Viswanath%27s_constant

http://www.maa.org/devlin/devlin_3_99.html

Class Notes.

Class Assignments.