Syllabus for M.A. Comprehensive/Ph.D. Qualifying Exam in Analysis
May/August 2006

John Albert

1. Topology of $\mathbf{R}^n$, uniform continuity, uniform convergence, the Riemann integral.

2. Functions of bounded variation, Jordan's theorem, the Riemann-Stieltjes integral.

3. Lebesgue outer measure on $\mathbf{R}^n$, sets of measure zero, the Cantor set and Cantor-Lebesgue function, Lebesgue measurable sets and Lebesgue measure, nonmeasurable sets.

4. Lebesgue measurable functions, Egorov's theorem, Lusin's theorem, convergence in measure.

5. Definition of Lebesgue integral, Tchebyshev's inequality, Fatou's lemma, Monotone Convergence Theorem, Dominated Convergence Theorem, expression of Lebesgue integral as Riemann-Stieltjes integral, equality of Lebesgue integral and Riemann integral for Riemann integrable functions, characterization of Riemann integrable functions.

6. Fubini's Theorem and Tonelli's Theorem on $\mathbf{R}^n$.

7. Vitali covering lemmas, Hardy-Littlewood maximal function, Lebesgue's theorem on derivative of indefinite integral in $\mathbf{R}^n$, derivatives of monotone functions, absolutely continuous and singular functions.

8. L^p and l^p spaces, Young's inequality, Hölder's inequality, Minkowski's inequality, metric spaces and Banach spaces, separability of L^p, Hilbert spaces, complete orthonormal bases for L^2, Bessel's inequality, Parseval's formula, Riesz-Fischer theorem on isometry between L^2 and l^2.

9. σ-algebras and measure spaces, additive set functions and measures, Jordan decomposition of additive set functions, integration with respect to a measure, absolutely continuous and singular set functions and measures.