Math 4513

1. *a.* Show that if a number n is not divisible by 3 then n^2 is also not divisible by 3.
b. Use part *a.* to show that $\sqrt{3}$ is irrational; i.e., there do not exist numbers m and n such that $3 = m^2/n^2$.

2. The Fibonacci numbers are defined by: $F_1 = 1$, $F_2 = 1$, and $F_{n+2} = F_{n+1} + F_n$ for all $n \geq 1$. The first few Fibonacci numbers are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...
a. Use the Euclidean algorithm to prove that the greatest common divisor of any two consecutive Fibonacci numbers is 1.
b. Use the Euclidean algorithm to find integers x and y such that $34x + 21y = 1$.

3. Use the fundamental theorem of arithmetic (i.e., the theorem on unique prime factorization) to prove that
a. $\sqrt{6}$ is irrational.
b. $\sqrt[3]{2}$ is irrational.