Let L_n stand for the maximum number of regions obtained when the plane is cut by n lines, E_n stand for the maximum number of regions obtained when space is cut by n planes, and P_n stand for the maximum number of regions (segments) obtained when a line is cut by n points. Obviously, P_n is equal to $n+1$. We arrived at some values of L_n and E_n in class, which are summarized in the table below:

n	P_n	L_n	E_n
0	1	1	1
1	2	2	2
2	3	4	4
3	4	7	8
4	5	11	15
5	6	16	

This table led us to a conjecture a couple of *recurrence relations* for the sequences L_n and E_n. These recurrence relations are equations relating given terms L_n and E_n in the sequences to preceding terms L_{n-1} and E_{n-1}.

We also surmised that the values in the above table are connected to the values in the following table, which is known as *Pascal's triangle*:

n	$\binom{n}{0}$	$\binom{n}{1}$	$\binom{n}{2}$	$\binom{n}{3}$	...
0	1	0	0	0	...
1	1	1	0	0	...
2	1	2	1	0	...
3	1	3	3	1	...
4	1	4	6	4	...
5	1	5	10	10	...

(You may remember from one of your previous math courses that there are simple formulas for the columns in Pascal's triangle. The first column is given by the formula $\binom{n}{0} = 1$, the second column is given by $\binom{n}{1} = n$, the third by $\binom{n}{2} = \frac{n(n-1)}{2}$, the fourth by $\binom{n}{3} = \frac{n(n-1)(n-2)}{2 \cdot 3}$, and so on.)

1. Write down a recurrence relation for L_n, and prove it using geometric arguments.

2. Guess a formula for L_n. (Hint: use the connection between L_n and Pascal's triangle). Prove that this guess is correct, using induction and the recurrence relation you gave in problem 1.

3. Write down a recurrence relation for E_n, and prove it using geometric arguments.

4. Guess a formula for E_n. (Again, you can find the formula by looking at the connection between E_n and Pascal's triangle.) Prove it using induction and the recurrence relation you gave in problem 3.