Answers to even-numbered problems in Chapter 3

Remember, the answers given below are just intended for you to use to check your work on the assignments. If you were to submit answers to these questions on a homework assignment, quiz, or exam, you would be expected to show the work you did to obtain the answers as well!

Asst. 4
3.1 #2: \( a = -2, b = -2, c = -5. \)

3.1 #12: (a) \( u + v = \begin{bmatrix} 0 \\ 8 \end{bmatrix}, \ u - v = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \ 2u = \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \ 3u - 2v = \begin{bmatrix} 10 \\ -1 \end{bmatrix}; \) (b) \( u + v = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \ u - v = \begin{bmatrix} -3 \\ 1 \end{bmatrix}, \ 2u = \begin{bmatrix} 0 \\ 6 \end{bmatrix}, \ 3u - 2v = \begin{bmatrix} -6 \\ 5 \end{bmatrix}; \) (c) \( u + v = \begin{bmatrix} 5 \\ 8 \end{bmatrix}, \ u - v = \begin{bmatrix} 0 \\ 8 \end{bmatrix}. \)

3.3 #6: (a) the set is a subspace (b) the set is not a subspace (c) the set is a subspace (d) the set is not a subspace.

3.3 #14: (a) the set is not a subspace (b) the set is a subspace (c) the set is a subspace.

Asst. 5
3.5 #2: (a) the set is not a basis (b) the set is not a basis (c) the set is a basis (d) the set is not a basis.

3.5 #26: (a) 2 (b) 3 (c) 3 (d) 3

Asst. 6

3.6 #2: \( x_1 = 2t - s, \ x_2 = s, \ x_3 = t, \) where \( s \) and \( t \) are any real numbers. Other parameterizations are also possible. (b) \( \mathbf{x} = s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}. \) Again, other answers are possible here.

3.6 #8: The only solution is \( x_1 = 0, \ x_2 = 0, \ x_3 = 0. \) So the solution space contains only one vector, \( \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \); i.e., it is the trivial vector space. It has no basis, and its dimension is zero (see Definition 3.11). This problem didn’t illustrate the process of finding the basis and dimension of a solution space very well. I should have assigned a problem like #7 in this section instead.

3.7 #2: \( [v]_S = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} \)

3.7 #22: I did this one in class. \( T = \{w_1, w_2, w_3\} \) where

\[
\begin{align*}
\mathbf{w}_1 &= \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \quad 
\mathbf{w}_2 &= \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad 
\mathbf{w}_3 &= \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}.
\end{align*}
\]
Asst. 7

3.8 #6: I did this one in class.

(a) \{[1 2 -1 3],[0 1 -5 9],[0 0 1 \ -\frac{5}{7}]\}. Other answers are possible.
(b) \{[1 2 -1 3],[3 5 2 0],[0 1 2 1]\}. Other answers are possible.

3.8 #12: (a) row rank = column rank = 3; (b) row rank = column rank = 2;
(c) row rank = column rank = 2.