Math 5453 — Fall 2012
Exam 2

(Note: In doing these problems, you may cite any result that we have done in class, without having to prove it.)

1. Show that if f and g are integrable on $\mathbf{R}$, then $\max\{f, g\}$ is integrable on $\mathbf{R}$.

2. Show that if f is integrable on $[0, \infty)$ then

a. $\lim_{n\to\infty} \int_0^\infty e^{-nx} f(x)\ dx = 0.$

b. $\lim_{n\to\infty} \int_n^\infty f(x)\ dx = 0.$

3. Suppose f is a non-negative function defined on a measurable set E. Prove that for every $\alpha > 0$,

$$m\left(\{x \in E : f(x) > \alpha\}\right) \leq \frac{1}{\alpha^2} \int_E f^2\ dx.$$

4. Let $\{f_n\}$ be a sequence of non-negative measurable functions on E such that $\int_E f_n(x)\ dx \leq 1$ for all n. Show that

$$\sum_{n=1}^{\infty} \frac{f_n(x)}{2^n}$$

is finite for almost every $x \in E$. Carefully justify the steps in your proof!

5. Let $\{f_n\}$ be a sequence of measurable functions on $[0, 1]$, and let $M > 0$ be such that $|f_n| \leq M$ on $[0, 1]$ for every n. Suppose that for every simple function h on $[0, 1]$,

$$\lim_{n\to\infty} \int_{[0,1]} h f_n\ dx = 0.$$

Prove that for every bounded measurable function g on $[0, 1]$,

$$\lim_{n\to\infty} \int_{[0,1]} g f_n\ dx = 0.$$