Math 5453 — Fall 2012
Exam 1

(Note: For these problems, you may cite any result that we have done in class, without having to prove it.)

1. If E_1 and E_2 are measurable sets, show that

$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2).$$

2. For $k > 0$ and $E \subseteq \mathbf{R}$, define $kE = \{y : y = kx,\ x \in E\}$. Show that $m^*(kE) = km^*(E)$.

3. Using the definition from problem **2**,

a. Show that if H is a subset of $\mathbf{R}$ of type G_δ, then kH is also of type G_δ.

b. Show that if E is a measurable set, then kE is measurable. (Hint: use part **a** and problem **2**.)

4. Show that there exist non-measurable sets A and B such that $A \cup B$ is measurable.

5. Suppose f is a measurable function on a measurable set E.

a. Show that if $f(x) > 0$ for all $x \in E$, then $1/f$ is measurable on E.

b. Show that if $f(x) \neq 0$ for all $x \in E$, then $1/f$ is measurable on E.