Math 5423 Review for Exam 1

The first exam will cover chapters 1 and 2 of the text. In the lectures we have followed the sequence of topics in the text pretty closely, which means that an easy way to review for the test is just to re-read these two chapters, and try some of the practice problems I posted on the course web page.

There are a couple of fairly minor differences between the presentation in the book and the lectures.

First, I didn't really cover Propositions 1.3.2 and 1.3.3, or Lemma 1.4.5; they're interesting, but not essential to the main discussion. More interesting for us than Lemma 1.4.5 is Corollary 1.5.2, which we did prove in class. Even more important is Theorem 1.5.3, which we also proved in class, although you might have a little trouble finding it in your notes because we didn't do it until after having gone some ways into Chapter 2.

Second, I skipped the discussion on pages 39 and 40 about the Jacobians of holomorphic maps, and the discussion on pages 48 to 50 in which the Cauchy integral theorem is related to Green's theorem. Those were presented in the text as different ways of looking at results that we had already proved by other methods. I did use Green's theorem a couple of times in the class as a short cut for evaluating some line integrals around closed curves. I won't expect you to know it for the exam, but it is good for you to be aware of it. (By the way, the statement of Green's Theorem on page 48 of the text is incorrect; there should be a plus sign instead of a minus sign in front of the Q in the integral on the left-hand side.)

Third, I skipped the explicit computations of Cauchy integrals on page 51 and 52, as these are again examples of obtaining a previously proved result (in this case, the Cauchy integral formula) by different methods.

Fourth, in class I took a slightly different approach to the "general form of the Cauchy integral formula and Cauchy integral theorem" than the one given in Section 2.6. This difference will not be relevant to any of the problems on the exam, though.

If you're wondering what you should have memorized for the exam, the list should certainly include the definition of holomorphic function on an open set (which entails knowing what the Cauchy-Riemann equations are), the definition of harmonic function, the definition of a complex line integral, the fundamental theorem of complex line integrals (Proposition 2.1.6), and the statements of the Cauchy integral formula and Cauchy integral theorem, at least in the simple versions of Theorems 2.4.2 and 2.4.3. Of course, if you've been working on the homework problems regularly, these definitions and theorems will mostly take care of themselves because you'll have absorbed them unconsciously by now!