Review for First Exam

The first exam (Wednesday, June 6) will cover Sections 2.1, 2.2, 2.3, 2.4, 2.5, 3.1 and 3.2 of the text. To review for the exam you should go over the class notes through Friday, June 1; and homework assignments 1 through 7. You can also use the review guide to the text below.

2.1. The algebraic and order properties of R. Much of this section is taken up with listing the algebraic and order axioms for **R** (subsections 2.1.1 and 2.1.5) and using them to prove some theorems about basic algebraic and order properties of the real numbers (2.1.2, 2.1.3, and 2.1.5 to 2.1.11). You are all familiar with these basic properties already, though you may not have seen them proved from the axioms before. I won't ask for proofs of simple facts like this on the exam.

In class we took a somewhat different approach from the text. We started by assuming only the algebraic and order properties of the integers, and explained how to define first the rational numbers, as ordered pairs, and then the real numbers, as Dedekind cuts. We didn't give all the details of this process, though. Anyway, there will not be any questions on the test over this material.

You should review the last subsection, titled "Inequalities", on pages 28 to 30. Being able to do things with inequalities like the examples in 2.1.12, and being familiar with useful inequalities like the ones in 2.1.13, will be necessary for doing some of the problems on this exam and on later exams as well. In particular, you should remember Bernoulli's inequality on page 30.

2.2. Absolute value and the real line. Like section 2.1, this section contains material that you've already seen in previous math classes. However, it presents a good opportunity to consolidate your knowledge of how to work with absolute values in inequalities, including knowledge not only of what you can do, but just as importantly, what you can't do. For example, you can't solve the inequality $|x+3| < |2x|$ by subtracting $|2x|$ from both sides to get $|3-x| < 0$.

2.3. The completeness property of R. First, make sure you have the definition of supremum memorized, since you can't solve any problems about supremums without knowing it. Next, go through the examples in 2.3.5, the examples in class, and the exercises from section 2.3 on assignments 2 and 3. Finally, the consensus of our class seemed to be that there weren't enough examples in the body of the text, so to make up for this you should pick a couple of exercises from section 2.3 that weren't assigned and try to do them. Even if you don't succeed the experience will be valuable.

In the text, the Completeness Property is an assumption or axiom we make about the real numbers, but in class, we stated it as a theorem which you can prove about the real numbers, once you've defined the real numbers as Dedekind cuts. For this exam, it doesn't matter which point of view you take. You should just make sure that you know what the Completeness Property says, and how to use it.

2.4. Applications of the supremum property. In class we went over the example in 2.4.1(b) (top of page 41), and the Archimedean Property 2.4.3 and its corollaries 2.4.4

and 2.4.5. I did not cover the rest of the material in this section, and you will not need to know it for this exam. We will come back later on in the course, though, to the Density Theorem 2.4.8 and its corollary, 2.4.9.

2.5. Intervals. In class we covered the subsections titled "Nested Intervals" and "The Uncountability of **R**" on pages 47 to 49. You will not need to know the rest of the material in section 2.5 for this exam.

3.1. Sequences and their limits. In class we covered all the material in this section except the subsection titled "Tails of Sequence" on page 59 and Example 3.1.11(d) on page 61.

We gave a slightly different version of Theorem 3.1.10 in class. Our version said that if (a_n) and (b_n) are sequences such that $|a_n| \leq b_n$ for every n, and $\lim b_n = 0$, then $\lim a_n = 0$. This is actually the same as Theorem 3.1.10, once you take into account the fact that a sequence (x_n) converges to x if and only if the sequence $(a_n) = (x_n - x)$ converges to zero.

You should make sure you have the definition of limit memorized (3.1.3).

Even though there were plenty of examples in this section, it would still be a good idea to try to do a couple of extra problems from the end of the section.

3.2. Limit theorems. For this exam, you should review Theorems 3.2.2 and 3.2.3, and Examples 3.2.8(a,b,c,d,e). You should also be aware of Theorems 3.2.9 and 3.2.10.

The proofs of Theorems 3.2.2 and 3.2.3 are instructive, and I encourage you to try to understand clearly how they work. A little bit of effort expended in this direction will be well worth it; it will make the course easier by helping you understand how all the ideas fit together.