Quiz 3

1. Give the definition of limit of a function at a point.

2. Give the definition of "f is continuous at c".

3. Give the definitions of "f is differentiable at c" and "derivative of f at c".

(NB: The "reasons" for your answers to the questions in #4 can range from rigorous proofs to intuitive explanations, but they should cite some valid general principle.)

4. State whether each sentence is true or false. Give reasons for your answers.

a. If $\lim\limits_{x\to 0} f(x) = 7$, then $f(0) = 7$.

False. (In class we said "the value of a function at a point has nothing to do with the limit of the function at the point".) An example showing the statement is false is: $f(x) = \begin{cases} 7 & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases}$.

b. If $\lim\limits_{x\to 0} f(x) = 7$, then $\lim f(1/n) = 7$.

True, by the sequential criterion for limits. In this case, since $\lim(\frac{1}{n}) = 0$ and $\frac{1}{n} \neq 0$ for all $n \in \mathbb{N}$, the sequential criterion tells us that $\lim\limits_{x\to 0} f(x) = 7$ implies $\lim(f(\frac{1}{n})) = 7$.

c. If f is continuous at 0 and $\lim\limits_{x\to 0} f(x) = 7$, then $f(0) = 7$.

True. By a theorem mentioned in class, if f is cont. at 0, then $\lim\limits_{x\to 0} f(x) = f(0)$. If $\lim\limits_{x\to 0} f(x) = 7$, it follows that $f(0) = 7$.

d. If f is continuous at 0 and $\lim\limits_{x\to 0} f(x) = 7$, then $\lim f(1/n) = 7$.

True, by the sequential criterion for continuity, which says that if f is continuous at 0 and $(x_n) = (\frac{1}{n})$, then $\lim(x_n) = 0$, so $\lim f(x_n) = f(0)$. In part c, we saw that since $\lim\limits_{x\to 0} f(x) = 7$, then $f(0) = 7$, so $\lim(f(x_n)) = 7$.