Math 4163
Assignment 2

1.
(a) Show that $\{\cos(n\pi x)\}_{n=0,1,2,3,\ldots}$ is an orthogonal system on $[0,1]$.
(b) Find the constants c_n which make $\{c_n \cos(n\pi x)\}_{n=0,1,2,3,\ldots}$ an orthonormal system. (Note: c_0 will not be the same as the rest of the c_n's.)
(c) Suppose you want to represent the function $f(x)$ as

$$f(x) = \sum_{n=0}^{\infty} A_n \cos(n\pi x).$$

By equating the inner product of both sides of this equation and using your answer to part (b), find formulas for the constants A_n.

2. The representation found in part (c) of the preceding problem is called the Fourier cosine series of $f(x)$ on $[0,1]$. Find the Fourier cosine series of the function $f(x) = x^2$ on $[0,1]$. Evaluate the constants A_n as explicitly as you can.

3. Verify that each of the functions $u_0(x,y) = y$ and $u_n(x,y) = \sinh(n\pi y)\cos(n\pi x)$ for $n = 1, 2, 3, \ldots$ satisfies the equation

$$u_{xx} + u_{yy} = 0$$

and the conditions

$$u_x(0,y) = u_x(1,y) = 0, \qquad u(x,0) = 0.$$

(These are called boundary conditions because they are conditions on the boundary of the unit square in the x-y plane.) Show that any linear combination

$$u(x,y) = A_0 y + \sum_{n=1}^{N} A_n \sinh(n\pi y)\cos(n\pi x)$$

satisfies the same differential equation and the same boundary conditions.