Math 4103
Review for Exam 3

The third exam will cover sections 29 – 34, 37 – 41, 44, 46, and 49 – 52 of the text. The relevant homework assignments are Assignments 8, 9, and 10. Below are some suggestions for how to review the material in the text (of course, I think you should review your notes from class as well), and a summary of what material from these sections you can skip.

29. The exponential function. Everything in this section was covered. The key facts are in equations (1), (2), (3), and (5), but everything in this section should be familiar to you.

30. The logarithmic function. Everything in this section was covered, and should be learned thoroughly. The function $\log z$ is the trickiest of the elementary functions to deal with, because it, like $\arg z$, is multi-valued. (In fact, $\log z$ is related to $\arg z$ through the formula $\log z = \ln |z| + i \arg z$.) Remember to distinguish between $\log z$ and its principal branch $\text{Log } z$. The latter is related to the principal branch of the argument function through the formula $\text{Log } z = \ln |z| + i \text{ Arg } z$.

31. Branches and derivatives of logarithms. We covered everything in this section. Besides re-reading it, take note of the fact that the material in this section was needed later on in section 44, when we integrated $1/z$ over the circle C_2 shown in Figure 51. A similar computation was done in class, when we integrated $1/z$ over a different portion of the circle, and used a different branch of the logarithm function.

32. Some identities involving logarithms. We covered from the beginning of the section through the first paragraph at the top of page 99. It wouldn't hurt to look at the rest of the section also, of course. One of the main lessons of this section is that the familiar identities from calculus about real-valued logarithms generally don't hold, or at least don't hold in the same form, for complex logarithms.

33. Complex exponents. You should re-read from the beginning of the section through Example 3. Again, the rest of the section is worth reading, too, and carries the message that the usual laws of exponents for real numbers don't always hold for complex exponents, which are generally multi-valued.

34. Trigonometric functions. We covered more or less the entire section. The basic formulas are (1) – (4) and (9). You don't need to memorize all the trigonometric identities, such as (5) – (8) or (10) – (14).

37, 38. Derivatives and integrals of functions $w(t)$. We covered these two sections in their entirety. Remember to distinguish between the derivatives and integrals in these sections, which are derivatives and integrals of complex-valued functions of a real variable, and the derivatives and integrals in most of the the rest of the book, which are derivatives and integrals of complex-valued functions of a complex variable. (Of course, when computing the integral of a complex function of a complex variable we often first reduce it to the integral of a complex function of a real variable; as in the examples in Section 41.)

39. Contours. We covered the entire section. It mostly just introduces some terminology, but (as you might suspect from the fact that I spent nearly an entire lecture on it in class), I think it's very important to get this terminology straight before trying to proceed with the theory and computations in later sections.

40. Contour integrals. 41. Some examples. In these two sections, contour integrals are defined and their definition is illustrated with examples. Understanding what's in these sections is basic to all the material that follows.

44. Antiderivatives. This section describes the technique you would usually use to calculate the integral of an analytic function over a contour which is not closed. It should be read completely. Notice that you have to take care sometimes to choose a branch of the antiderivative that is analytic on a domain containing the contour.

46. Cauchy-Goursat theorem. This is the basic theorem about integrals of complex functions over closed contours. Read the entire section.

49. Multiply connected domains. The theorem in this section is stated in more generality than the version I gave in class — it considers the case when there are several curves inside a big curve, not just one inside the other. Still, I recommend reading the entire section.

50. Cauchy integral formula. You should know the theorem in this section (you do not need to study its proof), and be able to do problems like the one in the example on page 164, or the ones on the homework assignments.

51. An extension of the Cauchy integral formula. You can start with formula (6) on page 167 (you do not need to know its proof), and read the two examples which follow.

52. Some consequences of the extension. All we covered from this section is the Corollary on p. 109. You can skip the rest.