Review for First Exam

The first exam covers the material in sections 2.1, 2.2, 2.3, 2.3, 2.4, 2.5, 3.1, 3.3, and 3.4 of the text. The golden section search is also covered on the exam. Sections 2.6 and 3.2 are not covered.

Below, "by hand" means using a calculator, but only for arithmetic, not for running programs. I will write the exam so that all you should need to do are simple additions and multiplications, but you might want to bring a calculator for those.

The exam will be closed book and closed notes.

2.1 Bisection method. You should know the bisection algorithm and be able to carry out a few iterations by hand. Read from the beginning of the section through Example 2.

2.2 Fixed-point iteration. Read page 56, and from page 60 through page 64. You do not need to know Theorem 2.3, though it is interesting. You should know Theorem 2.4 and its proof. In Corollary 2.4, estimate (2.5) is useful, but you don't need to know estimate (2.6) or its proof.

2.3 Newton's method and its extensions. Read from the beginning of the section through page 72. You should know the algorithms for Newton's method and the secant method, and be able to carry out a few steps of each method by hand. You do not need to know the proof of Theorem 2.6. You can skip the material on pages 73 to 75 on the method of false position.

2.4 Error analysis for iterative methods. Read from the beginning of the section through page 83. You can skip pages 84 and 85. You should study and understand the proofs of Theorem 2.8 and Theorem 2.9, but you don't need to memorize the proofs. Notice in particular the bullet point on page 82: if $f(p) = 0$ and $f'(p) \neq 0$, then for starting values sufficiently close to p, Newton's method will converge at least quadratically.

2.5 Accelerating convergence. You should review the whole section; however you do not need to memorize the algorithms for Aitken's Δ^2 method or Steffenson's method. If I ask a question involving either of these methods I will remind you in the question what the method is.

Golden section search. Recall that the golden section search for finding the minimum value for a unimodal function goes like this: start with an interval $[a, b]$. Pick two points c and d in $[a, b]$ using the formulas

$$c = a + (b - a)\lambda^2, \qquad d = a + (b - a)\lambda,$$

where $\lambda = \frac{\sqrt{5}-1}{2} = 0.618\ldots$. Compute $f(c)$ and $f(d)$. If $f(c) > f(d)$, the minimum is between c and b, so we take our new interval $[a_1, b_1]$ to be $[c, b]$. If $f(c) < f(d)$, the minimum is between a and d, so we take our new interval $[a_1, b_1]$ to be $[a, d]$. Then we repeat the process, starting with $[a_1, b_1]$ instead of $[a, b]$.

You should understand and be able to use this algorithm, and know the error bound for it (which is that after n steps, $|p_n - p| \leq \lambda^n(b - a)$.

3.1 Interpolation and the Lagrange Polynomial Read from "Lagrange Interpolating Polynomials" on page 108 to the end of this section. You can skip the proof of Theorem 3.3. You should be able to do a problem like Example 2 or Example 3 from this section.

3.3 Divided differences. Read from the beginning of the section through page 127. You should be able to do a problem like Example 1 in this section (only with simpler numbers, of course).

3.4 Hermite interpolation. Read Definition 3.8. You can skip Theorem 3.9 and its proof, and skip Example 1. Start reading from "Hermite Polynomials Using Divided Differences" on page 139, and read through Example 2. You should be able to use divided differences to do a problem like those in exercise 1 at the end of section 3.4 (answers to this problem are in the back of the book).