Math 2934 — Summer 2015
Review for third exam

The third exam will be on sections 15.1, 15.2, 15.3, 15.4, 15.7, 15.8, 15.9, 16.1, and 16.2 of the text. This material was covered on Assignments 8 through 13.

15.1, 15.2. These sections were already covered on the second exam, but I'm including them here because they're closely related to, and necessary for, the rest of chapters 15 and 16. Reviewing the definition of the double integral in section 15.1 helps with understanding the definition of the triple integral in section 15.7, which in turn helps you to understand why the formulas for integrals in cylindrical and spherical coordinates are the way they are.

More specifically, if you think of an integral as a sum of products of functions times the areas of little squares or the volumes of little cubes, then you realize that to get integrals in other coordinate systems correct, you need to figure out the areas of little squares or the volumes of little cubes in those coordinate systems.

15.3. Double integrals over general regions. We covered the entire section. Examples 3, 4, and 5 are particularly basic.

15.4. Double integrals in polar coordinates. We covered the entire section.

15.7. Triple integrals. We covered the entire section. However, you don't need to know the formulas for moments and centers of mass on page 1047. I won't ask about these on the exam.

15.8, 15.9. Triple integrals in cylindrical and spherical coordinates. We covered these sections in their entirety.

16.1. Vector fields. You should know what a *vector field* in $\mathbf{R}^2$ or $\mathbf{R}^3$ is (the definitions are on page 1081), and you should know what a *conservative* vector field is, and what the *potential function* for a conservative vector field is (see page 1085). Otherwise the material in this section is not essential.

16.2. Line integrals. There are actually two different kinds of line integral discussed in this section, and we only covered one kind in class.

The first kind of line integral, which we did not cover in class, is a line integral with respect to arc length, denoted in the book by the symbol $\int_C f\ ds$. You can skip the material in this section about line integrals with respect to arc length.

That means that you only need to read the material about the second kind of line integral, so you can start reading at the box numbered 7 near the bottom of page 1090, and go from there to the end of the section (skipping Example 5 if you like).

You will see that the text mentions "line integrals with respect to x or y (or z)" and "line integrals of vector fields". These are actually the same thing. The line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ of a vector field

$$\mathbf{F} = P(x, y, z)\ \mathbf{i} + Q(x, y, z)\ \mathbf{j} + R(x, y, z)\ \mathbf{k}$$

is the same as the integral

$$\int_C P(x, y, z)\ dx + Q(x, y, z)\ dy + R(x, y, z)\ dz,$$

which could also be called a "line integral with respect to x, y, and z". To evaluate such an integral, we must fix a parametrization of the curve C by giving x, y, and z along the curve as functions $x(t)$, $y(t)$, and $z(t)$ of a parameter t, for some range of values of t given by $a \leq t \leq b$. Then, as explained in class, we replace $P(x, y, z)$, $Q(x, y, z)$, and $R(x, y, z)$ with functions of t obtained by substituting these functions $x(t)$, $y(t)$, and $z(t)$ for x, y, and z; and we replace dx by $x'(t)\ dt$, dy by $y'(t)\ dt$, and dz by $z'(t)\ dt$. Finally we perform the integral with respect to t from $t = a$ to $t = b$.