

Name: Key

1. For the function $f(x, y) = x^2y + \sin(xy)$, find:

[2] a) f_x $2xy + y \cos(xy)$

[2] b) f_{xx} $2y + (y^2)(-\sin(xy))$ ~~$2y - y^2 \sin(xy)$~~

[2] c) f_{xy} $2x + (-yx) \cos(xy) + 1 \cdot \cos(xy)$

[2] d) f_y $x^2 + x \cos(xy)$

[2] e) f_{yx} $2x - xy \cos(xy) + 1 \cdot \cos(xy)$

2. For the surface $z = \frac{2+x}{3-y}$,

a) find the equation of the tangent plane to the surface at $P(1, 2, 3)$.

[6] $\frac{\partial z}{\partial x} = \frac{1}{3-y} \Rightarrow \frac{\partial z}{\partial x} \Big|_P = \frac{1}{3-2} = 1$; $\frac{\partial z}{\partial y} = (2+x)(-1)(3-y)^{-2} \cdot (-1)$
 $\Rightarrow \frac{\partial z}{\partial y} \Big|_P = (2+1)(3-2)^{-2} = 3$;

so $(z-3) = 1 \cdot (x-1) + 3(y-2)$

[4] b) use the tangent plane to find an approximate value of z when $x = 0.998$ and $y = 2.003$.

② $z - 3 \approx (0.998 - 1) + 3(2.003 - 2)$

$z - 3 \approx -0.002 + 0.009$

$z - 3 \approx 0.007$

② $z \approx 3.007$