Calculus II — Review for third exam

The second exam covers (most of) chapter 11 of the text. The relevant homework assignments are assignments 10 through 13.

Here is a brief guide to the sections of the text covered on the exam.

11.1. This section mostly introduces definitions. I won't ask for any definitions on this exam, but there are a few that you should know in order to understand the remainder of the chapter. These are the following. A *sequence* is just a list of numbers, in which each number is associated to a natural number n which indicates its position in the sequence. The number associated to the nth position in the sequence is usually indicated by a letter with the subscript n, such as a_n. A sequence is *decreasing* if each number is greater than or equal to the number which follows it, or in other words $a_n \geq a_{n+1}$ for all $n = 1, 2, 3 \ldots$. A sequence is *bounded above* if there exists some number M which is greater than every term of the sequence. A sequence can have a *limit*: the precise definition of what the limit of a sequence is involves epsilons and deltas, but an intuitive definition is all you need to know for now. Intuitively, to say that the sequence has a limit L means that the numbers in the sequence come arbitrarily close to L as n goes to infinity. An important basic fact about sequences is that if a sequence is increasing and bounded above, then it must have a limit.

11.2. Distinguish carefully between a sequence of numbers a_1, a_2, a_3, $\ldots$ and the *series* formed by this sequence. This series is denoted by $\sum a_n$, and actually is also a sequence, but is obtained by adding the numbers of the original sequence a_n one at a time. Thus we actually define the series $\sum a_n$ to be the sequence s_n whose terms are given by

$$\begin{aligned} s_1 &= a_1, \\ s_2 &= a_1 + a_2, \\ s_3 &= a_1 + a_2 + a_3, \\ s_4 &= a_1 + a_2 + a_3 + a_4, \end{aligned}$$

and so on. The *sum* of the series (also called the *infinite sum*) is by definition the limit of the sequence s_1, s_2, s_3, $\ldots$. This limit might not exist, however. If the limit exists, we say the series is convergent; if not, we say the series is divergent.

Thus, for example, the sequence $1/n$ does have a limit, but the series $\sum 1/n$ does not. The limit of $1/n$ is zero, but the series $\sum 1/n$ diverges to infinity.

An important kind of series is a *geometric series*, which is defined on page 730. You should know when a geometric series diverges and when it converges, and in the latter case you should know the formula given on page 730 for its sum.

There is an important fact about series in the section: if a series $\sum a_n$ converges, then the limit of the sequence a_n must be zero. (Another way of saying the same thing is that if the limit if the sequence a_n is not zero, then the series $\sum a_n$ cannot converge.) This statement has a simple and instructive proof (at the top of page 733). I won't ask for the proof on this exam, though.

11.3. You should know the Integral Test (page 740). In particular, you should know which series it applies to (series $\sum a_n$ in which the terms a_n are non-negative, decreasing, and have limit zero) — and which series it doesn't apply to!

For this exam, you don't need to know the material on pages 742 and 743 about estimating the remainder of series using the Integral Test. Also, I won't be asking for the proof of the Integral Test.

11.4. You should know the Comparison Test (page 746) and the Limit Comparison Test (page 748). You don't need to know the material on pages 749 and 750 about estimating sums via the comparison test. I also won't ask for the proof of the comparison test, but I recommend that you read it anyway (top of page 747) — it's simple, and understanding the proof of something makes it easier to remember and use.

In particular, if you have read and understood the proof of the Comparison Test, you'll be less likely to make the most common mistake students make when using it, which is, when given a series, finding a smaller series which converges, or a larger series which diverges, and then try to conclude something about the original series. You can't! Finding a smaller series which converges or a larger series which diverges does not tell you whether the original series converges or diverges.

11.5. You should know the Alternating Series Test (page 751). You don't need to know its proof. I might also ask a question about estimating sums for alternating series, like the one in Example 4, or the ones in problems 21– 30 at the end of the section.

A common misconception is that you can use the Alternating Series Test to prove that a series diverges. This is incorrect. There exist alternating series which do not satisfy the hypotheses of the Alternating Series Test because the absolute values of the terms don't decrease, but which still converge. However, if you are looking at an alternating series in which the limit of the terms is not zero, then the series must diverge — not because of the the Alternating Series Test, but because of the fact mentioned above in the discussion of Section 11.2.

11.6 You should know the meanings of the terms *absolutely convergent* and *conditionally convergent*. See examples 1, 2, 3 from this section, and the problems on Quiz 6.

You should know the Ratio Test (top of page 758). Notice that you can never use the Ratio Test to prove that a series is conditionally convergent. If the Ratio Test applies, it will either tell you that your series is absolutely convergent or divergent. If your series (unbeknownst to you) is conditionally convergent, the Ratio Test will give you an inconclusive answer, and you'll still be in the dark.

You won't need to know the material on page 761 about rearrangements, though it is interesting.

11.7 I'm not convinced a discussion like the one in this section is useful to students, but go ahead and look at it if you like. I suppose it can't hurt.

11.8. You should read everything in this section, on power series and their intervals of convergence, carefully.

11.9. Examples 1, 2, 3, 5, 6, and 7 are worth reading.

11.10. You should know the form of the *Taylor series* for a function (box 6 on page 778) and the *Maclaurin series* (box 7), which is a special case of a Taylor series. You do not need to know the material on pages 779 – 781 about convergence of Taylor series. You should be able to find Taylor series or Maclaurin series for functions, as in the examples in this section and on the homework.

11.11. This section is not covered on the exam.