Calculus II — Review for second exam

The second exam covers sections 7.1, 7.2, 7.3, 7.4, 7.8, 8.1, 8.2, and 8.3 of the text. The relevant homework assignments are assignments 5 through 9.

There won't be questions on this exam asking for definitions or proofs. I do recommend that you memorize the basic integral formulas: the formulas for the integrals of u^n (including u^{-1}), e^u, $\sin u$, $\cos u$, $\sec^2 u$, $\sec u \tan u$, $\tan u$, $\sec u$, $\dfrac{1}{\sqrt{1-u^2}}$, and $\dfrac{1}{1+u^2}$. (These are collected in the text as formulas number 2, 3, 4, 6, 7, 8, 10, 12, 14, 16, and 17 on "Reference Page 6" near the end of the book.) Remember, the reason for memorizing these is to learn certain patterns that enable you to figure out which substitutions to make in order to do more complicated integrals. Even if you had a table of integrals available, trying to do complicated integrals without having the basic ones memorized would would be like trying to write a poem in a language you don't understand, armed only with a dictionary.

At first there might seem to be a lot of material covered on this test. But if you focus on understanding one or two typical examples from each section, you shouldn't have much difficulty with whatever comes your way. To see if you've reached this level of understanding you might pick a random homework problem from each section that you haven't done yet, and see if you can work it without too much trouble.

Here is a brief guide to the sections of the text covered on the exam.

7.1. You should memorize the integration by parts formula, in the red box numbered "2" on page 488. (Notice there is another version of this formula, for definite integrals, in the red box numbered "6" on page 491.) Review Examples 1 through 5 in this section, the problems in Assignment 5, and problem number 1 on Quiz 3. (The answers to Quizzes 3 and 4 have been posted on the course web page.)

7.2. It's not necessary to memorize the long list of strategies in the red boxes on pages 497 and 498. They only work on some, not all, of the problems in this section anyway. What I recommend doing instead is reading through these strategies (without trying to memorize them) and noticing how each one is used in the examples of this section. For example, strategy (a) on page 497 is illustrated in Example 1 on page 496.

All these strategies are just simple integrations by substitution. The only thing that makes the problems in these section different from other problems on integration by substitution is that here, you have to make appropriate use of the basic trig identities $\sin^2 x + \cos^2 x = 1$ and $\tan^2 x + 1 = \sec^2 x$, and sometimes also the half-angle formulas listed in strategy (c) on page 497. You don't have to memorize the half-angle formulas; I'll supply them on the exam for any problem in which you need them. I do strongly recommend memorizing the formulas $\sin^2 x + \cos^2 x = 1$ and $\tan^2 x + 1 = \sec^2 x$, though.

7.3. You should review the entire section. The integrals in this section are actually fairly easy to do once you grasp the recipe for doing them. I recommend that you learn the following by heart:

If $\sqrt{a^2 - x^2}$ appears in the integral, put $x = a \sin\theta$;
if $\sqrt{a^2 + x^2}$ appears in the integral, put $x = a \tan\theta$;
if $\sqrt{x^2 - z^2}$ appears in the integral, put $x = a \sec\theta$.

These are easy to remember if you know the basic trig identities $\sin^2\theta + \cos^2\theta = 1$ and $\tan^2\theta + 1 = \sec^2\theta$.

The recipe for doing the integrals in this section is: first make the substitution indicated by the above rule, then integrate the resulting trigonometric integral using the appropriate substitution as in section **7.2**, then convert your answer back into a function of x. To help with this last step, draw a triangle showing the correct relationship between x and θ. For example, if in the first step you put $x = 5\tan\theta$, then $\tan\theta = \dfrac{x}{5}$, so you should draw a triangle in which the opposite side from angle θ has length marked "x" and the adjacent side has length marked "5".

7.4. Review Examples 1 through 6. There won't be a question on the test with repeated irreducible quadratic factors in the denominator, so you needn't worry about Examples 7 and 8. Example 9 is interesting, though, and you might review it just to improve your general skill at integration by substitution.

7.8. You should review the entire section. We haven't had any quiz questions on this section yet, so you should probably pay special attention to reviewing it.

Notice that, although there won't be questions on this exam asking for the definition of the words "convergent" and "divergent", you will still need to know what these words mean.

The comparison test on page 549 takes some getting used to. Look carefully at Examples 9 and 10, and problems 49 to 54.

8.1, 8.2, 8.3. For this exam you don't have to memorize the formulas for arc length, surface area, moments, or centroids. If one of the exam questions asks about these, it will include the needed formula. For example, a question about surface area of a solid of revolution about the x-axis would include the formula $S = \displaystyle\int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$. However, you should still review the material to make sure you know how to use these formulas. Look at Examples 1 and 2 from Section 8.1; Examples 1, 2, and 3 from Section 8.2; and Examples 4, 5, and 6 from Section 8.3; and review the problems from Assignment 9.