Calculus II — Review for first exam

The exam covers sections 6.1, 6.2, 6.3, 6.4, 6.6, and 6.8 of the text. The relevant homework assignments are Assignments 1 through 4.

You should know the following for the exam:

• The definition of e: in class, we defined the number e as

$$e = \lim_{h \to 0} (1+h)^{1/h}.$$

But you can give a different definition if you like (as long as it's correct).

• The proof that the derivative of e^x is e^x.

For this proof you're allowed to use the fact that $\lim_{h \to 0} \dfrac{e^h - 1}{h} = 1$. The proof goes like this:

$$\begin{aligned} \frac{d}{dx}(e^x) &= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} \\ &= \lim_{h \to 0} e^x \left(\frac{e^h - 1}{h} \right) \\ &= e^x \lim_{h \to 0} \frac{e^h - 1}{h} = e^x \cdot 1 = e^x \end{aligned}$$

You do not have to reproduce this proof exactly, as long as your proof includes all the main steps.

• The proofs that the derivative of $\ln x$ is $1/x$ (see pages 410–11 of the text), the derivative of $\arcsin x$ is $\dfrac{1}{\sqrt{1-x^2}}$ (bottom of page 454, starting with the sentence "Let $y = \sin^{-1} x$"), and the derivative of $\arctan x$ is $\dfrac{1}{1+x^2}$ (page 457, right after Example 4).

Here is a brief guide to the sections covered on the exam.

6.1. I didn't assign homework from this section, but it's worth briefly reviewing for background knowledge. The main thing in this section that you might not have seen before is the formula in Theorem 7, for the derivative of an inverse function. You won't need to know this formula for the exam, but you should be aware that it exists (essentially it says that if y is a function of x, then $dy/dx = \dfrac{1}{dx/dy}$.)

6.2. You should be very familiar with the basic properties of exponential functions covered on pages 391 to 394; you will need them frequently. The most basic of all are the laws of exponents at the bottom of page 393. You should also review everything else in this section, except you can skip the "Applications of exponential functions" on pages 394–395.

6.3, 6.4, 6.6. Review these sections in their entirety; you should know them quite well. This would also be a good time to make sure you have memorized the basic differentiation and integration formulas: from the table of differentiation rules in the back of the book (Reference page 5) these are #1 through 8, 9, 10, 11, 13, 14, 15, 17, 19, and 21; and from the table of integrals (Reference page 6) numbers #1 through 8, 10, 16, and 17. (Actually you really only need to memorize the integration formulas #16 and 17 with $a = 1$, as long as you know how to apply them to more general integrals using integration by substitution.)

6.8. There may be one or two problems on the exam about L'hopital's rule, but only easy ones — by which I mean only problems in which the limit is already in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$.