Quiz 7

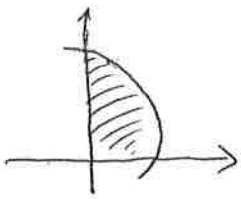
Name: Solution

[6] **1.** Find the slope of the tangent to the curve $x = 3t + \sin t$, $y = t^3 + e^t$ at the point where $t = 0$.

① $\frac{dx}{dt} = 3 + \cos t$ ① $\frac{dy}{dt} = 3t^2 + e^t$ ② $\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{1}{4}$

① $\left.\frac{dx}{dt}\right|_{t=0} = 3 + 1 = 4$ ① $\left.\frac{dy}{dt}\right|_{t=0} = 1$

[6] **2.** Find the area enclosed by the polar curve $r = e^{3\theta}$ between $\theta = 0$ and $\theta = \frac{\pi}{2}$ (see diagram).



① ① ① ①

$$A = \frac{1}{2}\int_0^{\pi/2} e^{6\theta}\, d\theta = \frac{1}{12} e^{6\theta}\Big]_0^{\pi/2}$$

① ①

$$= \frac{1}{12}\left[e^{3\pi} - 1\right]$$

(using $A = \frac{1}{2}\int r^2 d\theta$) ② ①

[8] **3.** Find the arc length of the polar curve $r = e^{3\theta}$ between $\theta = 0$ and $\theta = \frac{\pi}{2}$.

$$L = \int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$\frac{dr}{d\theta} = 3e^{3\theta}$$ ①

$$\left(r^2 + \frac{dr}{d\theta}^2\right) = e^{6\theta} + 9e^{6\theta} = 10e^{6\theta}$$ ① ① ①

$$L = \int_0^{\pi/2} \sqrt{10 e^{6\theta}}\, d\theta = \sqrt{10}\int_0^{\pi/2} e^{3\theta}\, d\theta = \frac{\sqrt{10}}{3} e^{3\theta}\Big]_0^{\pi/2}$$ ① ① ①

$$= \frac{\sqrt{10}}{3}\left(e^{\frac{3\pi}{2}} - 1\right)$$ ①