Quiz 5

Name: key

[10] **1.** Use the integral test to determine whether the series $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ converges.

(Hint: be careful to change the limits in the integral if you use a substitution.)

$$\int_1^{\infty} \frac{e^{1/x}}{x^2}\,dx = \lim_{b\to\infty} \int_1^b \frac{e^{1/x}}{x^2}\,dx = \lim_{b\to\infty} \int_1^{1/b} -e^u\,du =$$

(1) (1)

$$\boxed{\begin{array}{l|l} u = 1/x \quad (1) & x = 1 \to u = 1 \quad (1) \\ du = -1/x^2\,dx \quad (1) & x = b \to u = 1/b \end{array}}$$

$$= \lim_{b\to\infty} \left[-e^u\right]_1^{1/b} = \lim_{b\to\infty} \left[-e^{1/b} - (-e^1)\right] = \lim_{b\to\infty} \left(-e^{1/b} + e\right) = -e^0 + e$$

(1)

$$= -1 + e.$$

Since $\int_1^{\infty} \frac{e^{1/x}}{x^2}\,dx$ converges (2), and $f(x) = \frac{e^{1/x}}{x^2}$ is decreasing* and $\to 0$ as $x\to\infty$, and is nonnegative, then $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ converges by the Integral Test (2)

* clearly $\frac{1}{x}$ and $\frac{1}{x^2}$ both decrease as x increases, so $e^{1/x} \cdot \frac{1}{x^2}$ does also.

[10] **2.** Use the comparison test to determine whether the series $\sum_{n=1}^{\infty} \frac{2^n}{3^n + \log n}$ converges. Give a complete explanation.

Let $a_n = \frac{2^n}{3^n + \log n}$ and $b_n = \frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n$. (2)

(2)

Since $3^n + \log n \geq 3^n$, then $\frac{1}{3^n + \log n} \leq \frac{1}{3^n}$, so $\frac{2^n}{3^n + \log n} \leq \frac{2^n}{3^n}$,

So $a_n \leq b_n$ for all $n \geq 1$. Also $0 \leq a_n$ for all n.

(2)

Since $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ is a geometric series (2) with ratio $r = \frac{2}{3}$ (less than 1), then $\sum b_n$ converges.

So by the Comparison Test, $\sum a_n$ converges.

(2)