Quiz 4

Name: key

1. Evaluate the integral $\int \frac{1}{\sqrt{x^2+1}}\,dx$. Simplify your answer, if possible.

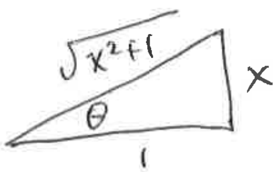
[10]

(You may want to use the fact that $\int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta|$.)

Put $\boxed{\begin{aligned} x &= \tan\theta \\ dx &= \sec^2\theta\,d\theta \end{aligned}}$ Then $\int \frac{1}{\sqrt{x^2+1}}\,dx = \int \frac{1}{\sqrt{\tan^2\theta+1}}\sec^2\theta\,d\theta =$

$= \int \frac{1}{\sec\theta}\cdot\sec^2\theta\,d\theta = \int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta|+C$

From the triangle:



we see that if $\tan\theta = x$, then $\sec\theta = \sqrt{x^2+1}$.

So the answer is $\boxed{\ln\left|x+\sqrt{x^2+1}\right|+C}$

2. Evaluate the integral $\int \frac{7x+11}{x^2+3x+2}\,dx$.

Put $\frac{7x+11}{x^2+3x+2} = \frac{7x+11}{(x+2)(x+1)} = \frac{A}{x+1} + \frac{B}{x+2}$.

Then $7x+11 = A(x+2)+B(x+1)$.

Putting $x=-1$, we get $-7+11 = A(-1+2)+0$, or $A=4$.

Putting $x=-2$ we get $-14+11 = 0+B(-2+1)$, or $-B=-3$

or $B=3$.

So $\int \frac{7x+11}{x^2+3x+2}\,dx = \int \frac{4}{x+1}\,dx + \int \frac{3}{x+2}\,dx = \int \frac{4}{u}\,du + \int \frac{3}{w}\,dw$

$u = x+1$, $du = dx$; $w = x+2$, $dw = dx$

$= 4\ln|u| + 3\ln|w| + C$

$= \boxed{4\ln|x+1| + 3\ln|x+2| + C}$