EXAM 1
Math 2513
9-21-12

Name key

1. *(20 points)* Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent. (One way to do this is with a truth table.)

p	q	r	$p \wedge q$	$p \rightarrow r$	$q \rightarrow r$	$(p \wedge q) \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	T	T
T	F	F	F	F	T	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	F	T	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Since the circled entries in the columns for $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not the same, these two propositions are not equivalent

2. *(15 points)* Consider the statement

$$\forall n \exists m (n < m)$$

and assume that the domain of the variables consists of all natural numbers $\mathbb{N}$.

a) Express the statement in English.

For every natural number n, there is a natural number m such that $m > n$.

b) Is the statement true or false? Justify your answer.

True. For a given n, we can take $m = n+1$. Then $m > n$.

Now for parts c) and d), consider the statement

$$\exists m \forall n (n < m)$$

and assume again that the domain consists of all natural numbers.

c) Express the statement in English.

There exists a natural number m which is greater than every other natural number n.

d) Is the statement true or false? Justify your answer.

False. No matter what natural number m is given, there is always another natural number n (for example $n = m+1$) such that m is not greater than n.

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3. *(20 points)* Show that if $n^2 + 3$ is not divisible by 4, then n is even. Use a proof by contraposition.

Suppose n is odd. Then
$\exists k \in \mathbb{N} \quad n = 2k+1$.

So
$$n^2+3 = (2k+1)^2+3$$
$$n^2+3 = (4k^2+4k+1)+3$$
$$n^2+3 = 4k^2+4k+4$$
$$n^2+3 = 4(k^2+k+1).$$
Since $k^2+k+1 \in \mathbb{N}$, this shows that n^2+3 is divisible by 4. //

4. *(15 points)* Show that if A, B, and C are sets, then

$$(A-B)-C \subseteq A-(B-C).$$

1st method: If $x \in (A-B)-C$, then $x \in A-B$ and $x \notin C$. Since $x \in A-B$, then $x \in A$ and $x \notin B$. Since $x \notin B$, then $x \notin B-C$ (because x can only be in $B-C$ if x is in B). So $x \in A$ and $x \notin B-C$. So $x \in A-(B-C)$.

2nd method: Use a membership table:

A	B	C	$A-B$	$(A-B)-C$	$B-C$	$A-(B-C)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	1	0
0	1	1	0	0	0	0
1	0	0	1	1	0	1
1	0	1	1	0	0	1
1	1	0	0	0	1	0
1	1	1	0	0	0	0

In every row where $(A-B)-C$ is 1, we see that $A-(B-C)$ is also 1 (there is only one row where this happens, see the circled entries.)
So $(A-B)-C \subseteq A-(B-C)$.

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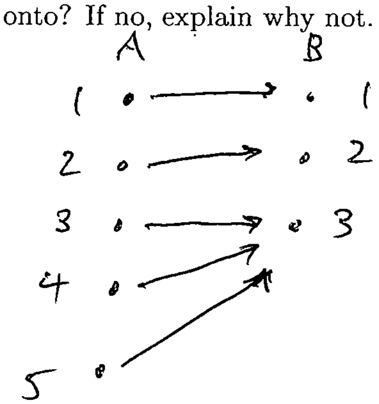
5. *(15 points)* Suppose $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3\}$.

a) Does there exist a function $f : A \to B$ which is one-to-one? If no, explain why not. If yes, give an example.

No. If f were one-to-one, then it would assign each element of A to a different element of B. Then the image of f in B would have to contain 5 different elements, one for each element of A. Since B has only 3 elements, this is impossible. //

b) Does there exist a function $f : A \to B$ which is onto? If no, explain why not. If yes, give an example.

Yes. For example:



6. *(15 points)* You are on an island where all the inhabitants are either knights, who only tell the truth, or knaves, who only tell lies. You meet three inhabitants: A, B, and C. A says, "B is a knave." B says, "Either A and C are both knights, or A and C are both knaves."

Is C a knight or a knave?

(Remember that to get credit for the answer, you must explain why your answer has to be correct.)

We will prove that C is a knave.

There are two cases to consider: either A is a knight or A is a knave.

If A is a knight, then his statement "B is a knave" is the truth, so B is a knave. So B's statement is false, which means A and C cannot both be the same thing. Since A is a knight, this means C must be a knave.

If A is a knave, then his statement is false, so B is a knight. So B is telling the truth when he says A and C are both the same. Since A is a knave, this means C must be a knave also. //