Math 2513
Review for Final Exam
Dec. 11, 2012, 8:00 a.m.

The final exam is comprehensive, covering all the material from the first three exams, together with the material covered in class since the third exam. You can use the review sheets for the first three exams to review for the material in sections 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 2.1, 2.2, 2.3, 2.4, 4.1, 4.2, 4.3, 5.1, 5.2, and 6.1 of the text. After the third exam, we covered portions of sections 6.2, 6.3, 6.5, 9.1, and 9.5. Below is a review guide to these sections.

As a general rule, especially for the material in Chapter 6, it doesn't help much just to memorize the formulas that appear. The basic skill to master is to be able to figure out, for a given problem, which formula applies to it. Really the only way to acquire this skill is to do a variety of problems thoughtfully. By "thoughtfully" I mean making a good effort to do the problem by yourself, and when you get the wrong answer being sure to figure out exactly why you went wrong, and learn from it.

6.2. The Pigeonhole Principle. The pigeonhole principle is easy to state (see Theorem 1 on page 399 and Theorem 2 on page 401), but applying it is sometimes tricky. I tried doing Example 10 or Example 11 on page 403 by myself (without looking at the solutions first) and had quite a bit of trouble. That would be a good exercise for you too. The proof of Theorem 3 is also tricky. Probably the best way to review this section is to try a good selection of exercises from the end of the section and from the supplemental exercises at the end of the chapter. When you write up a solution to a problem using the pigeonhole principle, try to identify what corresponds to the boxes and what corresponds to the objects being put into the boxes. For example, in problem 34, the boxes are giant boxes labelled with numbers from 0 to 8,008,278, and the objects are the people who live in New York City. Each person gets put into the box whose label is the same number as the number of hairs on that person's head.

6.3. Permutations and Combinations. We covered this entire section. It contains two different formulas: a formula for permutations in Theorem 1, and a formula for combinations in Theorem 2. Both formulas give the number of ways to choose r objects from a set of n distinguishable objects, when repetition is not allowed, or in other words you are not allowed to choose the same object twice. (Here "distinguishable" means that you can tell the objects apart. Two people are distinguishable, for example, but two letter A's are not.) The difference between the formulas is that one, the formula for permutations, applies when the order in which you make the r choices matters, while the other, the formula for combinations, applies when it does not matter what order you make the choices in.

Formulas for the number of ways to choose r objects from n objects when repetition is allowed, and for the number of ways to choose r objects from n objects when some of the objects are indistinguishable from each other, are covered in Section 6.5.

6.5. Generalized Permutations and Combinations. We covered the material in this section from the beginning through Example 9. This includes the formulas in Theorem 1 and 2 for the number of ways to choose r objects from n distinguishable objects when repetition is allowed, and when either the order of choices does matter (permutations with repetition) or the order of choices does not matter (combinations with repetition). In the case of combinations with repetition, the formulas is obtained from a clever method using "stars and bars".

The formula in Theorem 3 gives the number of ways to choose r objects from a set of n objects, with repetition not allowed, when some of these objects are indistinguishable from each other. In class, we called this type of problem a MISSISSIPPI problem, since an example is the problem of finding the number of ways to rearrange the letters in the word MISSISSIPPI.

From the subsection titled "Distributing Objects into Boxes", you can skip the material on problems involved in putting objects into indistinguishable boxes. Such problems are generally rather hard (no simple formulas exist for their solution), and we did not cover them in class. So from this subsection you need only review problems involving putting objects into distinguishable boxes: that is, Example 8, Theorem 4, and Example 9.

9.1. Relations and Their Properties. We discussed relations earlier in the semester, briefly in Section 2.1 (see page 124). Also remember that functions, which we discussed at length in Section 2.3, are special kinds of relations, and sequences, which we discussed in Section 2.4, are special kinds of functions.

Here in Section 9.1 we turn back to relations in general, which are not necessarily functions, and define three important properties: reflexivity, symmetry, and transitivity (Definitions 3, 4, and 5). There's also antisymmetry, which is a bit less important, but does come up from time to time. You should review the material in this section from the beginning through Example 15. You can skip the last subsection titled "Combining Relations".

9.5. Equivalence Relations. The whole section is worth reviewing.