

Quiz 7

Name: bey

1. A surface is parametrized by the equations

$$x = \cos u, \quad y = \sin u, \quad z = v$$

for $0 \leq u \leq 2\pi$ and $0 \leq v \leq 3$. Find:

(3) a. $\mathbf{r}(u, v) = (\cos u)\mathbf{i} + (\sin u)\mathbf{j} + v\mathbf{k}$ ($= x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$)

(3) b. $\mathbf{r}_u = (-\sin u)\mathbf{i} + (\cos u)\mathbf{j} + 0\mathbf{k}$

(3) c. $\mathbf{r}_v = 0\mathbf{i} + 0\mathbf{j} + 1\mathbf{k}$

(3) d. $\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\sin u & \cos u & 0 \\ 0 & 0 & 1 \end{vmatrix} = (\cos u - 0)\mathbf{i} - (-\sin u - 0)\mathbf{j} + 0\mathbf{k}$
 $= \cos u\mathbf{i} + \sin u\mathbf{j} + 0\mathbf{k}$

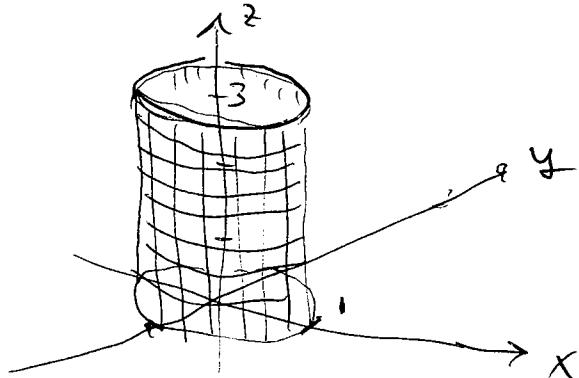
(3) e. $|\mathbf{r}_u \times \mathbf{r}_v| = \sqrt{\cos^2 u + \sin^2 u + 0^2} = \sqrt{1} = 1.$

- (4) f. $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$

$$= \int_0^{2\pi} \int_0^3 (x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}) \cdot (\mathbf{r}_u \times \mathbf{r}_v) \, dv \, du$$

$$= \int_0^{2\pi} \int_0^3 (\cos u\mathbf{i} + \sin u\mathbf{j} + v^2\mathbf{k}) \cdot (\cos u\mathbf{i} + \sin u\mathbf{j} + 0\mathbf{k}) \, dv \, du =$$

- (2) g. Sketch a picture of the surface in xyz-space:



$$\begin{aligned} &= \int_0^{2\pi} \int_0^3 (\cos^2 u + \sin^2 u) \, dv \, du \\ &= \int_0^{2\pi} \int_0^3 1 \, dv \, du \\ &= \int_0^{2\pi} 3 \, du = \boxed{6\pi} \end{aligned}$$