Quiz 6

Name: Key

[6]

1. Find the first three terms of the Maclaurin series for $\frac{x}{\sqrt{1+x}}$. (The easiest way is by using the binomial theorem.)

$$\frac{1}{\sqrt{1+x}} = (1+x)^{-\frac{1}{2}} = 1 + \left(-\tfrac{1}{2}\right)x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}x^2 + \dots$$

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$$\frac{x}{\sqrt{1+x}} = x\left(1 + \left(-\tfrac{1}{2}\right)x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}x^2 + \dots\right)$$

$$= x - \frac{x^2}{2} + \frac{3}{8}x^3 + \dots$$

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2. a. Evaluate the indefinite integral $\int \sin(x^2)\,dx$ as an infinite series.

2 $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$, so

2 $\sin(x^2) = x^2 - x^6/3! + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$, and

2 $\int \sin(x^2)dx = x^3/3 - (x^7/7)\cdot\frac{1}{3!} + (x^{11}/11)\cdot\frac{1}{5!} - (x^{15}/15)\cdot\frac{1}{7!} + \dots$

b. Use the first two nonzero terms of the series in **a** to get an approximate value for $\int_0^1 \sin(x^2)\,dx$.

2 $\int_0^1 \sin(x^2)dx \approx \left[\frac{x^3}{3} - \frac{x^7}{42}\right]_0^1 = \frac{1}{3} - \frac{1}{42} = \frac{13}{42}$

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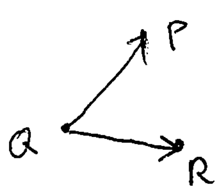
3. The points $P(1,2,3)$, $Q(4,3,8)$, and $R(-1,3,4)$ form a triangle in $\mathbf{R}^3$. Decide whether angle $\angle PQR$ is a right angle, giving a reason for your answer.



We have to check whether $\overrightarrow{QP}\cdot\overrightarrow{QR} = 0$.

Since $\overrightarrow{QP} = \langle 1-4, 2-3, 3-8\rangle = \langle -3,-1,-5\rangle$ 1

and $\overrightarrow{QR} = \langle -1-(4), 3-3, 4-8\rangle = \langle -5,0,-4\rangle$, 1

Then $\overrightarrow{QP}\cdot\overrightarrow{QR} = (-3)(-5) + (-1)\cdot 0 + (-5)(-4) = 15 + 0 + 20 = 35$ 2

Since $\overrightarrow{QP}\cdot\overrightarrow{QR} \neq 0$, $\angle PQR$ is not a right angle. 2

(Note: I mistakenly asked for $\angle PQR$; I had meant to ask for $\angle QPR$, which is a right angle.)