Quiz 1

Name: key

1. A curve is given parametrically by the equations $x = t^2 + t$, $y = t^2 + 2t - 1$.

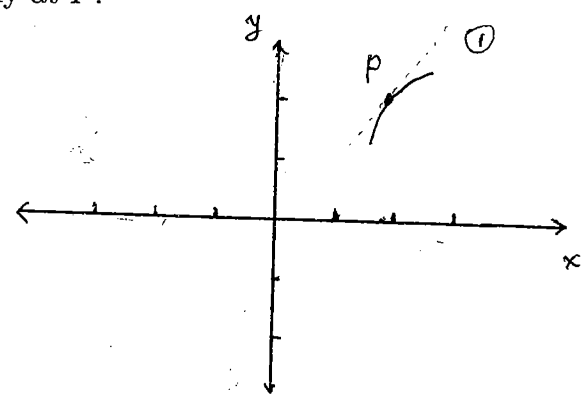
[4] **a.** Find dy/dx. $= \dfrac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \dfrac{2t+2}{2t+1}$

[5] **b.** Find d^2y/dx^2. $= \dfrac{d}{dx}(y') = \dfrac{\frac{d}{dt}(y')}{\frac{dx}{dt}} = \dfrac{\frac{d}{dt}\left(\frac{2t+2}{2t+1}\right)}{(2t+1)} = \dfrac{\left[\frac{(2t+1)\cdot 2 - (2t+2)\cdot 2}{(2t+1)^2}\right]}{(2t+1)}$

$= \dfrac{(4t+2)-(4t+4)}{(2t+1)^3} = \dfrac{-2}{(2t+1)^3}$

[4] **c.** Sketch the graph in the vicinity of the point P where $t = 1$. Draw the slope and concavity correctly at P.

When $t=1$, $x = 2$ and $y = 2$



$\left.\dfrac{dy}{dx}\right|_{t=1} = \dfrac{2\cdot 1+2}{2\cdot 1+1} = \dfrac{4}{3}$ (slope upward)

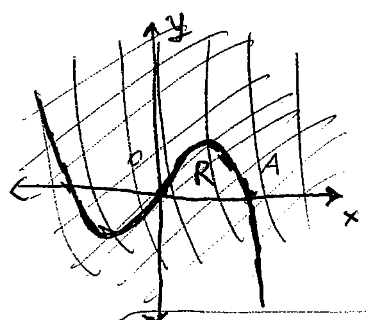
$\left.\dfrac{d^2y}{dx^2}\right|_{t=1} = \dfrac{-2}{3^3} = \dfrac{-2}{27}$ (concave down)

[7] **2.** The graph below shows a curve whose parametric equations are $x = t + t^2$ and $y = t - t^3$. Find the area of the region R in the first quadrant between the graph and the x-axis.

$A = \int_{t=0}^{t=1} y\,dx = \int_0^1 (t-t^3)(1+2t)\,dt$

$= \int_0^1 (t + 2t^2 - t^3 - 2t^4)\,dt$

$= \left[\dfrac{t^2}{2} + \dfrac{2t^3}{3} - \dfrac{t^4}{4} - \dfrac{2t^5}{5}\right]_0^1 = \dfrac{1}{2} + \dfrac{2}{3} - \dfrac{1}{4} - \dfrac{2}{5} = \dfrac{31}{60}$

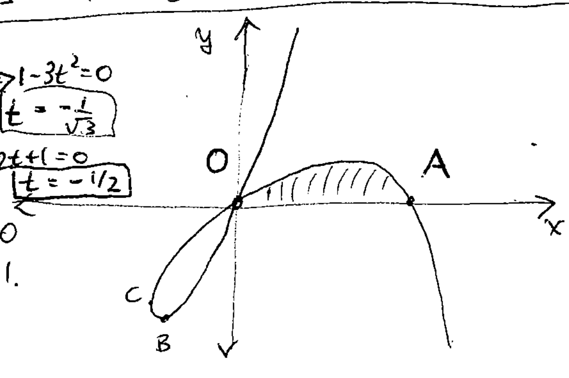


Extra Credit:
At B, $\frac{dy}{dt} = 0 \Rightarrow 1 - 3t^2 = 0 \Rightarrow \boxed{t = -\frac{1}{\sqrt{3}}}$

At C, $\frac{dx}{dt} = 0 \Rightarrow 2t+1 = 0 \Rightarrow \boxed{t = -1/2}$

At origin, $t=0$ and $t=-1$.

At A, $t=1$



At O and A, $y = 0$, so $t - t^3 = 0$, so $t(1-t^2) = 0$, so $t = 0$, 1 or -1.
But when $t=0$, $x = 0$
and when $t = 1$, $x = 2$
and when $t = -1$, $x = 0$.