Quiz 5

Name key Row ______

1. Find the limits using L'hospital's rule.

[4] **a.** $\lim_{x\to 0} \frac{\arctan x}{x} \underset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 0} \frac{\left(\frac{1}{1+x^2}\right)}{1} = \frac{\frac{1}{1+0^2}}{1} = \boxed{1}$

(2) (2)

[5] **b.** $\lim_{x\to 1} \frac{1-x+\ln x}{(x-1)^2} \underset{\left(\frac{1-1+\ln 1}{0^2} = \frac{0}{0}\right)}{=} \lim_{x\to 0} \frac{0-1+\frac{1}{x}}{2(x-1)} \underset{\left(\frac{-1+\frac{1}{1}}{2\cdot 0} = \frac{0}{0}\right)}{=} \lim_{x\to 0} \frac{0-\frac{1}{x^2}}{2(1-0)} = \frac{-\frac{1}{1^2}}{2\cdot 1} = \boxed{\frac{-1}{2}}.$

(2) (2) (1)

2. Evaluate the integrals, showing all work.

[5] **a.** $\int \frac{x}{\sqrt{1-x^4}}\,dx$ (Hint: let $u = x^2$.) $= \int \frac{\cancel{x}}{\sqrt{1-(x^2)^2}} \frac{du}{2\cancel{x}} = \int \frac{1}{\sqrt{1-u^2}} \frac{du}{2}$ (1)

$u = x^2$ (1)
$du = 2x\,dx$ (1)
$\frac{du}{2x} = dx$

$= \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\arcsin(u) + C$ (1)

$= \boxed{\frac{1}{2}\arcsin(x^2) + C}$ (1)

[6] **b.** $\int xe^{3x}\,dx = uv - \int v\,du$

$u = x$ $dv = e^{3x}\,dx$ (1)
(1) $du = dx$ $v = \frac{e^{3x}}{3}$ (1)

$= x\cdot\frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\,dx$ (1)

$= \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\,dx$

$w = 3x$
$dw = 3\,dx$

$= \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{w}\frac{dw}{3}$

$= \frac{1}{3}xe^{3x} - \frac{1}{9}\int e^{w}\,dw = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{w}$ (2)

$= \boxed{\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C}$