Find the volumes of the solids obtained by revolving the shaded regions around the $y$-axis. You need not simplify your answer.

1. The region between the curve $y = x^3 + x$, the line $x = 1$, and the $x$-axis.

\[
V = \int_{0}^{1} 2\pi y \, dx \quad \text{(0)}
\]
\[
= \int_{0}^{1} 2\pi x(x^3 + x) \, dx \quad \text{(0)}
\]
\[
= 2\pi \int_{0}^{1} (x^4 + x^2) \, dx \quad \text{(1)}
\]
\[
= 2\pi \left[ \frac{x^5}{5} + \frac{x^3}{3} \right]_{0}^{1} = 2\pi \left[ \frac{1}{5} + \frac{1}{3} \right] = 2\pi \cdot \frac{8}{15} = \frac{16\pi}{15}
\]

2. The region between the curve $x = y^3 + y$, the line $x = 2$, and the $x$-axis.

\[
V = \int_{0}^{2} \pi (x)^2 - \pi (y^3 + y)^2 \, dy \quad \text{(1)}
\]
\[
= \pi \int_{0}^{2} \left\{ 4 - (y^6 + 2y^4 + y^2) \right\} \, dy \quad \text{(2)}
\]
\[
= \pi \left[ 4y - \frac{y^7}{7} - \frac{2y^5}{5} - \frac{y^3}{3} \right]_{0}^{1}
\]
\[
= \pi \left[ 4 - \frac{1}{7} - \frac{2}{5} - \frac{1}{3} \right] = \frac{328\pi}{105}
\]