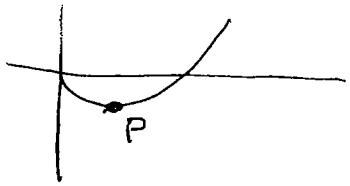
Quiz 5

Name: Key

1. The point $P(x, y)$ is the lowest point on the graph of $y = x^5 \ln x$ (see diagram). Find x and y. Simplify your answers.

$x = e^{-1/5}$

$y = -\frac{1}{5e}$



$\frac{dy}{dx} = x^5 \cdot \frac{1}{x} + 5x^4 \ln x$

$\frac{dy}{dx} = x^4 + 5x^4 \ln x$

$\frac{dy}{dx} = x^4(1 + 5\ln x)$. → So $\frac{dy}{dx} = 0$ when $1 + 5\ln x = 0$,

or when $\ln x = -\frac{1}{5}$, or $\boxed{x = e^{(-1/5)}}$ Then $y = x^5 \ln x$

$= (e^{-1/5})^5 \cdot \ln(e^{-1/5})$

$= e^{-1} \cdot (-\frac{1}{5}) = \boxed{\frac{-1}{5e}}$

2. Use L'hopital's rule to find $\lim_{x \to 0} \frac{\ln \sec x}{x^2}$.

$\lim_{x \to 0} \frac{\ln(\sec x)}{x^2} = \lim_{x \to 0} \frac{\frac{1}{\sec x}(\sec x \tan x)}{2x} = \lim_{x \to 0} \frac{\tan x}{2x} = \lim_{x \to 0} \frac{\sec^2 x}{2} = \frac{\sec^2 0}{2}$

$\left(\frac{\ln(\sec 0)}{0^2} = \frac{\ln(1)}{0} = \frac{0}{0}\right)$ $\left(\frac{\tan 0}{2 \cdot 0} = \frac{0}{0}\right)$

$= \boxed{\frac{1}{2}}$

3. Evaluate the integral $\int_0^1 \frac{e^x}{\sqrt{1 - e^{2x}}}\, dx$. Simplify your answer as much as possible.

Put $\begin{cases} u = e^x \\ du = e^x dx \end{cases}$. Then $\int_0^1 \frac{e^x}{\sqrt{1 - e^{2x}}} dx = \int_1^e \frac{du}{\sqrt{1 - u^2}}$

When $x = 0$, $u = e^0 = 1$

When $x = 1$, $u = e^1 = e$

$= \left[\arcsin(u)\right]_{u=1}^{u=e}$

$= \arcsin(e) - \arcsin(1)$

$= \boxed{\arcsin(e) - \frac{\pi}{2}}$