Quiz 4

Name: Answer key

[5] **1.** The spring on a bumping post in a freight yard is compressed 1 ~~inch~~ foot by a force of 36,000 pounds. Find the work done in compressing it 1/2 ~~inch~~ foot.

For a spring, $F = k \cdot x$, Here $36{,}000 = k \cdot 1$, so $k = 36{,}000$. ①

Hence, $F = 36{,}000\,x$. ①

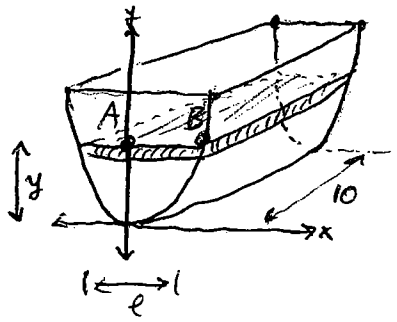
Work done in compressing $\frac{1}{2}$ foot $= \int_0^{1/2} F\,dx = \int_0^{\frac{1}{2}} (36{,}000)x\,dx$ ①

$= 36{,}000 \int_0^{\frac{1}{2}} x\,dx = (36{,}000)\left(\frac{1}{2}x^2\right)_0^{\frac{1}{2}} = (36{,}000)\cdot\frac{1}{8}$ ②

$= \boxed{4500 \text{ ft-lbs.}}$

[5] **2.** (This easy question is just *part* of a tank-pumping problem — no integration is involved here.)
The end of a tank has the shape of a parabola given by $y = x^2$, and the tank is 10 meters long. A horizontal layer of water Δy inches thick is located y meters above the bottom of the tank. (See diagram.)

a) Find the volume of the layer in cubic meters, as a function of y and Δy.



$AB = x = \sqrt{y}$

$\ell = 2(AB) = 2\sqrt{y}$ ②

volume $= \ell \cdot w \cdot h = (2\sqrt{y})(10)\cdot\Delta y$

$= (20)\sqrt{y}\,\Delta y$ ①

b) Find the weight of the layer in Newtons. (Water has a mass of 1000 kilograms per cubic meter, and the acceleration of gravity is 9.8 Newtons per kilogram.) ②

weight $= (9.8)(\text{mass}) = (9.8)(1000)(\text{volume}) = (9.8)(1000)(2\sqrt{y})(10)\Delta y$

$= (196{,}000)\sqrt{y}\,\Delta y$

[5] **3.** Differentiate: $\frac{d}{dx}\left(\frac{e^x}{e^{2x}+1}\right)$

$= \dfrac{(e^{2x}+1)\frac{d}{dx}(e^x) - e^x\,\frac{d}{dx}(e^{2x}+1)}{(e^{2x}+1)^2}$ ②

$= \dfrac{e^{2x+1}\cdot e^x - e^x\cdot(2e^{2x})}{(e^{2x}+1)^2}$ ① ②

4. Integrate: $\int_0^1 e^{5x}\,dx$

$= \int_0^5 e^u \frac{du}{5}$ ①

$= \frac{1}{5}\int_0^5 e^u\,du$

① $u = 5x$

$du = 5\,dx$

① $x = 0 \Rightarrow u = 0$

$x = 1 \Rightarrow u = 5$

$= \frac{1}{5}\left[e^u\right]_0^5$ ①

$= \frac{1}{5}\left[e^5 - e^0\right]$

$= \frac{1}{5}\left[e^5 - 1\right]$. ①