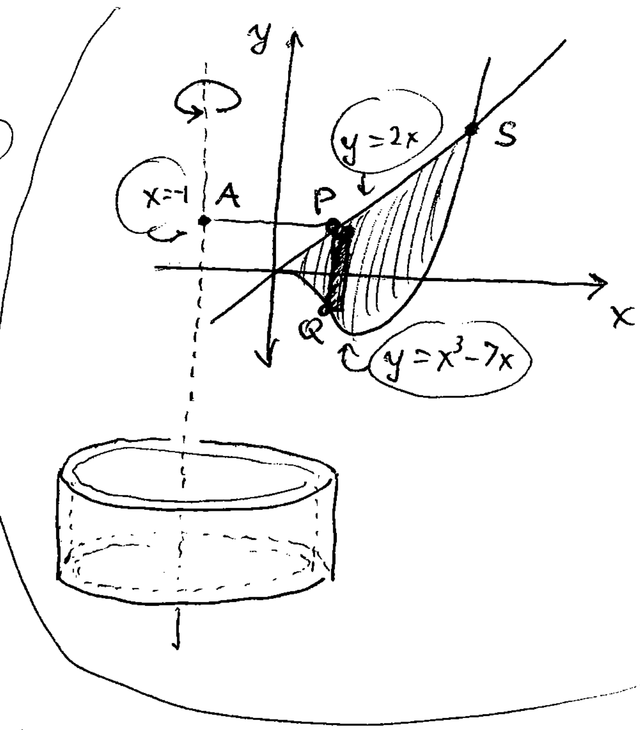
Quiz 3

Name: answer key

The shaded region in the diagram lies to the right of the y-axis, between the line $y = 2x$ and the curve $y = x^3 - 7x$. Find the volume of the solid obtained by rotating the region around the line $x = -1$.



The element of integration is a shell with height $h = PQ$ (1) and radius $r = AP$ (1) and thickness Δx. (1)

The volume of the shell is approximately $2\pi r h \Delta x$. (2)

The rightmost shell is at S, whose coordinates are found by setting $2x = y = x^3 - 7x$. This gives $x^3 - 9x = 0$ so $x^3 = 9x$, so $x^2 = 9$, so $x = \pm 3$. Here $x = 3$. (3)

The leftmost shell is at $x = 0$.

So the volume of the solid is (2) $V = \int_{x=0}^{x=3} 2\pi r h\, dx =$

$$= \int_0^3 2\pi \left(2x - (x^3 - 7x)\right)(x+1)\, dx$$

(2) (2)

(since $r = AP = x + 1$, $h = PQ = 2x - (x^3 - 7x)$)

$$= \int_0^3 2\pi (9x - x^3)(x+1)\, dx$$

(2)

$$= 2\pi \int_0^3 (9x^2 - x^4 + 9x - x^3)\, dx =$$

(2) (2)

$$= 2\pi \left[3x^3 - \frac{x^5}{5} + \frac{9x^2}{2} - \frac{x^4}{4} \right]_0^3 = 2\pi \left[3\cdot 3^3 - \frac{3^5}{5} + \frac{9\cdot 3^2}{2} - \frac{3^4}{4} \right]$$

$$= 2\pi \left[3^4 - \frac{3^5}{5} + \frac{3^4}{2} - \frac{3^4}{4} \right] = 2\pi\left[81\left(1 + \tfrac{1}{2} - \tfrac{1}{4}\right) - 81 \cdot \tfrac{3}{5} \right] = (2\pi)(81)\left(\tfrac{13}{20}\right) = \frac{1053\pi}{10}$$