answer key

Quiz 2

Name: answer key

1. Evaluate the integral $\int_{-4}^{1} \frac{x}{\sqrt{x+8}}\, dx$.

[7]

$\boxed{\begin{aligned} u &= x+8 \\ du &= dx \end{aligned}} \rightarrow \boxed{x = u-8}$

$\boxed{\begin{aligned} &\text{when } x=-4,\ u=4 \\ &\text{when } x=1,\ u=9 \end{aligned}}$

$$= \int_{u=4}^{u=9} \frac{u-8}{\sqrt{u}}\, du = \int_4^9 \frac{u}{\sqrt{u}} - \frac{8}{\sqrt{u}}\, du$$

$$= \int_4^9 u^{1/2} - 8u^{-1/2}\, du$$

$$= \left[\tfrac{2}{3} u^{3/2} - 8 \cdot 2u^{1/2} \right]_4^9$$

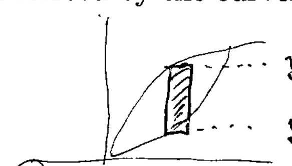
$$= \left(\tfrac{2}{3} 9^{3/2} - 16 \cdot 9^{1/2} \right) - \left(\tfrac{2}{3} 4^{3/2} - 16 \cdot 4^{1/2} \right)$$

$$= \left(\tfrac{2}{3} \cdot 27 - 16 \cdot 3 \right) - \left(\tfrac{2}{3} \cdot 8 - 16 \cdot 2 \right) = \tfrac{2}{3} \cdot 19 - 16 = \tfrac{38}{3} - \tfrac{48}{3}$$

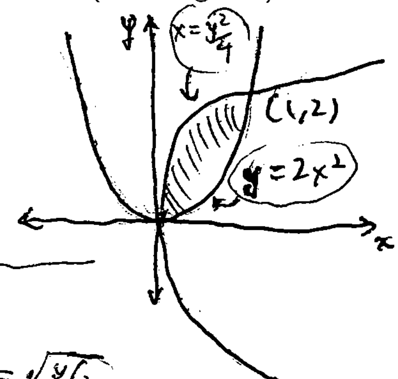
$$= \boxed{-10/3}$$

[7] **2.** Find the area of the region enclosed by the curves $x = y^2/4$ and $y = 2x^2$ (see diagram).

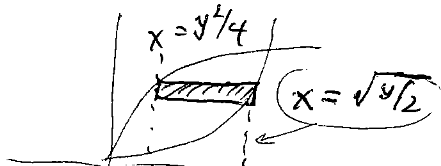
$$A = \int_{x=0}^{x=1} 2\sqrt{x} - 2x^2\, dx$$



$$= \left[2 \cdot \tfrac{2}{3} x^{3/2} - \tfrac{2x^3}{3} \right]_0^1 = \left(\tfrac{4}{3} - \tfrac{2}{3} \right) - (0 - 0) = \boxed{\tfrac{2}{3}}$$

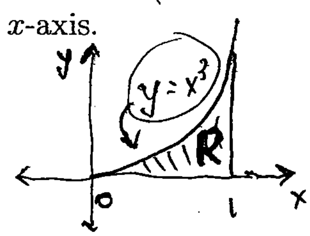


OR: $$A = \int_{y=0}^{y=2} \left(\sqrt{\tfrac{y}{2}} - \tfrac{y^2}{4} \right) dy$$



$$= \left[\tfrac{1}{\sqrt{2}} \tfrac{2}{3} y^{3/2} - \tfrac{y^3}{12} \right]_0^2 = \tfrac{1}{\sqrt{2}} \cdot \tfrac{2}{3} \cdot 2\sqrt{2} - \tfrac{8}{12} = \tfrac{4}{3} - \tfrac{2}{3} = \boxed{2/3}$$

[6] **3.** The region R lies between the curve $y = x^3$, the line $y = 0$, and the line $x = 1$ (see diagram). Find the volume of the solid obtained by rotating R around the x-axis.



$$V = \int_0^1 \pi y^2\, dx$$

$$= \int_0^1 \pi (x^3)^2\, dx$$

$$= \int_0^1 \pi x^6\, dx = \left[\frac{\pi x^7}{7} \right]_0^1 = \frac{\pi}{7}(1 - 0) = \boxed{\frac{\pi}{7}}$$