Linear Algebra, Spring 2016
Quiz 9 Name: _____

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Is there a linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^2$ such that

$$L\left(\begin{bmatrix}1\\2\\3\end{bmatrix}\right) = \begin{bmatrix}-1\\2\end{bmatrix}, \quad L\left(\begin{bmatrix}4\\5\\6\end{bmatrix}\right) = \begin{bmatrix}2\\3\end{bmatrix}, \quad L\left(\begin{bmatrix}7\\8\\9\end{bmatrix}\right) = \begin{bmatrix}6\\5\end{bmatrix}?$$

If so, find the matrix representation of L (with respect to the standard bases on $\mathbb{R}^3$ and $\mathbb{R}^2$). If not, explain why not.

L does not exist because:

$$\begin{bmatrix}7\\8\\9\end{bmatrix} = -\begin{bmatrix}1\\2\\3\end{bmatrix} + 2\begin{bmatrix}4\\5\\6\end{bmatrix}$$

So L would need to satisfy

$$L\left(\begin{bmatrix}7\\8\\9\end{bmatrix}\right) = L\left(-\begin{bmatrix}1\\2\\3\end{bmatrix} + 2\begin{bmatrix}4\\5\\6\end{bmatrix}\right)$$

$$= -L\left(\begin{bmatrix}1\\2\\3\end{bmatrix}\right) + 2L\left(\begin{bmatrix}4\\5\\6\end{bmatrix}\right)$$

$$= -\begin{bmatrix}-1\\2\end{bmatrix} + 2\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix} \neq \begin{bmatrix}6\\5\end{bmatrix}$$