Linear Algebra, Spring 2016
Quiz 7

Name: SOLUTIONS

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Let $\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}$ be a basis of a vector space V. Circle True or False for each of the following statements, and explain why your answer is correct.

a) (True or False) The subspace

$$W = \text{span}\{\vec{v_1} + \vec{v_2}, 2\vec{v_1} + 3\vec{v_2}, \vec{v_1} - \vec{v_2}\},$$

is two dimensional.

True: $W = \text{span}\{\vec{v_1}, \vec{v_2}\}$, and $\vec{v_1}, \vec{v_2}$ are lin. ind.,

so $\dim W = 2$

b) (True or False) Let $\vec{w_1}, \vec{w_2}, \vec{w_3}, \vec{w_4}$ be vectors such that

$$[\vec{w_1}\, \vec{w_2}\, \vec{w_3}\, \vec{w_4}] = [\vec{v_1}\, \vec{v_2}\, \vec{v_3}\, \vec{v_4}]A,$$

where A is a 4x4 matrix. If the equation $A\vec{x} = \vec{0}$ has a nontrivial solution, then $\vec{w_1}, \vec{w_2}, \vec{w_3}, \vec{w_4}$ is a basis of V.

Let $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ be nonzero vector with $A\vec{x} = \vec{0}$. Then

$x_1\vec{w_1} + \cdots + x_4\vec{w_4} = [\vec{w_1}\ \vec{w_2}\ \vec{w_3}\ \vec{w_4}]\vec{x} = [\vec{v_1}\ \vec{v_2}\ \vec{v_3}\ \vec{v_4}]A\vec{x} =$

$= [\vec{v_1}\ \vec{v_2}\ \vec{v_3}\ \vec{v_4}]\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \vec{0}$, so $\vec{w_1}, \dots, \vec{w_4}$ are not lin. ind.

c) (True or False) The set $\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}\}$ is a subspace of V.

$\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}\}$ not closed under scalar multiplication,

for example does not contain $\vec{0} = 0 \cdot \vec{v_1}$