

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Let $S \subset \mathbb{R}^5$ be the set of vectors

$$S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 3 \\ -2 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Find a basis for the vector space $W = \text{span}S$. What is the dimension of W ?

$$\text{rref}(S) = \begin{bmatrix} \textcircled{1} & 0 & -2 & 0 \\ 0 & \textcircled{1} & 3 & 0 \\ 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So 1st, 2nd, 4th vectors of S give basis for W .

$$\text{So } \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\} \text{ is a basis}$$

$$\text{and } \dim W = 3$$