

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Let  $S \subset \mathbb{R}^5$  be the set of vectors

$$S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ -4 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 1 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Find a basis for the vector space  $W = \text{span}S$ . What is the dimension of  $W$ ?

$$\text{rref}(S) = \begin{bmatrix} \textcircled{1} & 0 & 1 & 0 \\ 0 & \textcircled{1} & -5 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$  1st, 2nd vectors of  $S$  give a basis

So  $\begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ -4 \\ 0 \\ 2 \end{bmatrix}$  is a basis for  $W$

and  $\dim W = 2$