

You must show all your work to receive credit. Calculators are allowed.

**Problem 1:** (3 points) For each of the following problems,  $V$  is a vector space and  $W$  is a subset of  $V$ . Determine whether or not  $W$  is a *subspace* of  $V$ , and explain why or why not.

a)  $V = \mathbb{R}^3$ ,  $W$  is a plane which does not intersect the  $z$ -axis.

No:  $\vec{0}$  not in  $W$

b)  $V = P$  (the set of all polynomials), and  $W = \{f \in V \mid f(5) = 0\}$ .

Yes:  $f, g \in W \Rightarrow (f+g)(5) = f(5) + g(5) = 0 \Rightarrow f+g \in W$   
 $f \in W, c \in \mathbb{R} \Rightarrow (cf)(5) = cf(5) = 0 \Rightarrow cf \in W$

c)  $V = M_{2,2}$  (2x2 matrices) and  $W = \{A \in V \mid A^T = A\}$ .

Yes:  $A, B \in W \Rightarrow (A+B)^T = A^T + B^T = A+B \Rightarrow A+B \in W$   
 $A \in W, c \in \mathbb{R} \Rightarrow (cA)^T = cA^T = cA \Rightarrow cA \in W$