

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Suppose A is a 3×3 matrix, and the 3×6 augmented matrix $[A|I_3]$ has some row operations performed on it so that the resulting matrix is

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 2 & 1 \\ 0 & 2 & 0 & -4 & -2 & 10 \\ 0 & 1 & -2 & -6 & -4 & 7 \end{bmatrix}.$$

Find $(A^T)^{-1}$.

Since $[A|I_3]$ and $\begin{bmatrix} 1 & 2 & 3 & 0 & 2 & 1 \\ 0 & 2 & 0 & -4 & -2 & 10 \\ 0 & 1 & -2 & -6 & -4 & 7 \end{bmatrix}$ are related by row operations, they have the same rref. Thus

$$\text{rref}[A|I_3] = \text{rref}\left(\begin{bmatrix} 1 & 2 & 3 & 0 & 2 & 1 \\ 0 & 2 & 0 & -4 & -2 & 10 \\ 0 & 1 & -2 & -6 & -4 & 7 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & -2 & -5 & -6 \\ 0 & 1 & 0 & -2 & -1 & 5 \\ 0 & 0 & 1 & 2 & 1.5 & -1 \end{bmatrix}$$

in general, $\text{rref}[A|I_3] = [I_3|A^{-1}]$

so $A^{-1} = \begin{bmatrix} -2 & -5 & -6 \\ -2 & -1 & 5 \\ 2 & 1.5 & -1 \end{bmatrix}$

and so $(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & -2 & 2 \\ -5 & -1 & 1.5 \\ -6 & 5 & -1 \end{bmatrix}$