

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Find the RREF of the augmented matrix associated to the following system. Make sure you write down the operations you use to get the RREF. Then solve the system.

$$\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 7 \\ x_5 = -7 \\ 2x_1 + 4x_2 - 2x_3 + 4x_4 + 6x_5 = -20 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 7 \\ 0 & 0 & 0 & 0 & 1 & -7 \\ 2 & 4 & -2 & 4 & 6 & -20 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 7 \\ 2 & 4 & -2 & 4 & 6 & -20 \\ 0 & 0 & 0 & 0 & 1 & -7 \end{bmatrix}$$

$$\xrightarrow{R_2 \rightarrow R_2 - 2R_1} \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 7 \\ 0 & 0 & 0 & 2 & 6 & -34 \\ 0 & 0 & 0 & 0 & 1 & -7 \end{bmatrix} \xrightarrow{R_2 \rightarrow \frac{1}{2}R_2} \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 3 & -17 \\ 0 & 0 & 0 & 0 & 1 & -7 \end{bmatrix}$$

$$\xrightarrow{R_2 \rightarrow R_2 - 3R_3} \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & -7 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 - R_2} \begin{bmatrix} 1 & 2 & -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & -7 \end{bmatrix}$$

RREF is $\begin{bmatrix} 1 & 2 & -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & -7 \end{bmatrix}$

x_2, x_3 free
 $x_4 = 4$
 $x_5 = -7$
 $x_1 = 3 - 2x_2 + x_3$

or

s, t parameters
 $x_2 = s, x_3 = t$
 $x_4 = 4$
 $x_5 = -7$
 $x_1 = 3 - 2s + t$