

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Find the eigenvalues and eigenspaces of the matrix

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$\begin{aligned} P_A(\lambda) &= \text{Det} \left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \right) = \text{Det} \begin{bmatrix} \lambda-3 & -1 \\ -1 & \lambda-3 \end{bmatrix} \\ &= (\lambda-3)^2 - 1 = \lambda^2 - 6\lambda + 8 = (\lambda-4)(\lambda-2) \end{aligned}$$

eigenvalues: 2, 4

$$E_2 = \text{null sp} \begin{bmatrix} 2-3 & -1 \\ -1 & 2-3 \end{bmatrix} = \text{null sp} \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} = \text{span} \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

$$E_4 = \text{null sp} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$